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AP Physics I

Two Dimensional Dynamics

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Review of One Dimensional Dynamics

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Topics to Review

This chapter assumes that you have already studied Dynamics in One Dimension - which describes how the motion of objects can be predicted by knowing the forces that act on them.

But, only motion along an x or y axis - or east and west - or north and south - or up and down - were considered.

Life is more complex than that.

Topics to Review

We're going to consider motion that is a combination of movement along an x axis and a y axis - like the children's toy "Etch a Sketch" - either ask your parents or look it up on the web. But here's a picture of the inside of one - the metal pen in the lower right quadrant moves along the x and y axes of the rods.

First, we'll review the basics of One Dimensional Dynamics.

Topics to Review

- Newton's Three Laws of Motion
- Inertial Reference Frames
- Mass and Weight
- Forces studied:
  - weight / gravity
  - normal force
  - tension
  - friction (kinetic and static)
- Drawing Free Body Diagrams
- Problem Solving

Newton's Laws of Motion

First Law: An object maintains its velocity (both speed and direction) unless acted upon by a non-zero net force. This is also known as the Law of Inertia.

This Law actually wasn't put in place until Galileo Galilei proposed it in the 17th century.

Why wasn't this obvious to people like Aristotle in 350 B.C. who thought that an object had to be pushed continually to move?

Demo

Demo

Lab

Newton's Laws of Motion

A casual observer sees objects move when someone (or something) is pushing or pulling it - and when they stop the push or pull force, the object stops moving. This backs up Aristotle's view.

So, the First Law really is counterintuitive.

Unless you think about the other force that's acting on the object being moved - the force of friction. This force always acts to oppose motion, so once you stop pulling or pushing the object, it will soon come to rest because of the friction force.

This is what Galileo and then Newton recognized and codified.

1 When the engines on a rocket ship in deep space (where the gravitational attraction of any planets or stars are negligible) are turned off, it will:

○ A slow down and eventually stop.

○ B stop immediately.

○ C turn right.

○ D move with a constant velocity.

○ E I need help

Answer

https://www.njctl.org/video/?v=nRUWIPvDFIM

2 If a book on the console between the driver and the passenger seats starts moving forward, the forward velocity of the car must have:

○ A decreased.

○ B increased.

○ C stayed constant.

○ D changed direction to the right.

○ E I need help

Answer

https://www.njctl.org/video/?v=P4mcbgB26OI

Explain, using Newton's First Law, why seatbelts should be used.

https://www.njctl.org/video/?v=rsY-47Mfqd8

Answer

Newton's Laws of Motion

While discussing the First Law, we found that it was necessary to consider all the forces on an object - the net force - to determine its motion.

Specifically, a pushing or pulling force and the unseen frictional force.

Newton added another idea to this - what if the same net force was applied to different objects - with different masses.

How would the motion of those objects compare?

https://www.njctl.org/video/?v=cTSOzl1YO2w

Newton's Laws of Motion

This one's a little easier - our experience shows that if you push a heavy object, and then push a light object, the light object will move quicker - its acceleration will be greater.

Our experience also tells us that if we push an object with a large force its acceleration with be greater than if we push it will a small force.

So, we now have the concepts of considering all the forces on an object and taking into account the mass of the object.

This leads to Newton's Second Law.

Newton's Laws of Motion

Second Law: The sum of the external forces on an object is directly proportional to the product of its mass and acceleration. Note that force and acceleration are vectors.

$$\Sigma\vec{F} = m\vec{a}$$

For such a simple equation, it hides a trap.

Any ideas? Think of what this equation tells you about the motion of the object.

Demo

Newton's Laws of Motion

$$\Sigma\vec{F} = m\vec{a}$$

The trap is that the net force does NOT tell you how the object is moving - that is, it does not give you any information about its velocity or displacement. It only tells you the object's acceleration - its change in velocity.

An object may be moving at 300 m/s, with zero external force on it, so it has zero acceleration - which just means it keeps a constant velocity of 300 m/s.

3 The acceleration of an object is directly proportional to:

○ A the position of the object.

○ B the net force on the object.

○ C the velocity of the object.

○ D the object's mass.

○ E I need help

Answer

https://www.njctl.org/video/?v=KwAndaEccSQ

4 A net force F acts on a mass m and produces an acceleration a. What acceleration results if a net force 4F acts on a mass 6m?

○ A 4a

○ B a/6

○ C 6a

○ D 2a/3

○ I need help

https://www.njctl.org/video/?v=WZ3Sr1U71ck

Answer

5 Multi-Correct: If the net force on an object is 0 N, what does it tell you about the object?

☐ Its velocity is changing.

☐ It is stationary.

☐ Its velocity is constant.

☐ It has zero acceleration.

☐E I need help

https://www.njctl.org/video/?v=TYf6CEN81L8

Answer

Newton's Laws of Motion

One more law. And this one has its own twist.

Consider sitting on a small cart with wheels next to a brick wall. You push on the wall with your feet.

Which way do you go and why? You're exerting a force on the wall - which doesn't move as it is built into the ground and there is a great deal of friction and cement holding it in place. That explains why the wall isn't moving in response to your force.

And you know that an object at rest (you) won't move without a net external force. You've exerted a force on the wall, but why are you moving?

https://www.njctl.org/video/?v=ZZWc0xU_gZM

Newton's Laws of Motion

The wall is pushing you back!

It doesn't look like it's doing anything, and we haven't covered this yet, but recall from your earlier science courses what makes up atoms.

Negatively charged electrons orbit a nucleus.

The wall's electrons are being pushed by the electrons in your shoe - and like charges repel (this will be covered in the Electric Charge portion of this course), so the wall's electrons are pushing you back.

And since you're not cemented into the ground, you accelerate as described by Newton's Second Law.

Newton's Laws of Motion

Third Law: Whenever one object exerts a force on a second object, the second object exerts an equal force on the first object in the opposite direction.

The two objects form an "action-reaction" pair. Since both forces are simultaneous, the designation of either as the "action" force is somewhat arbitrary.

It is important to note that the forces act on different objects - don't make the mistake of applying these two forces to the same object - or it would seem that nothing could ever move.

Demo

6 Multi-Correct: A large truck collides with a small car, inflicting a great deal of damage to the car. Which of the following is true about the collision?

☐ The force on the truck is greater than the force on the car.

☐ The force on the car is greater than the force on the truck.

☐ The force on the truck is the same magnitude as the force on the car.

☐ During the collision the truck has a smaller acceleration than the car.

☐ E I need help

https://www.njctl.org/video/?v=4TLrJg9tUM8

Answer

7 Action-reaction forces are:

- ○ A equal in magnitude and point in the same direction.
- ○ B equal in magnitude and point in opposite directions.
- ○ C cancel each other out for a net force on each object of zero.
- ○ D unequal in magnitude and point in opposite directions.
- ○ E I need help

Answer

https://www.njctl.org/video/?v=8WnrzwwCiJs

8 The Earth pulls downward on a pen with a force F. If F is the action force, what is the reaction force?

○ A The pen pulling upward on the earth with a force F.

○ B The table pushing down on the floor with a force F.

○ C The pen pushing down on the table with a force F.

○ D The table pushing up on the pen with a force F.

○ I need help

https://www.njctl.org/video/?v=4COlXPBjtBE

Answer

9 A student is doing a hand-stand. An action - reaction pair of forces is best described as:

○ A The student pushes down on the ground -
The ground pushes up on the student

○ B Gravity is pulling the student down -
The ground is pushing the student up

○ C Gravity is pulling the student down -
The student's arms push the student up

○ D The student's hands push down on the ground -
The students arms push the student up

○ E I need help

https://www.njctl.org/video/?v=x3Sv2oqWUmM

Answer

Reference Frames

Physics is based on observation and measurement.

In order to measure something, it has to be compared against something else.

Let's take an example to show the importance of choosing what this something else is.

https://www.njctl.org/video/?v=3UDIUvAIdoc

Reference Frames

A person is in a bus and walked from the back of the bus to the front in 5 s. Another person sitting in the bus had a tape measure and measured how far he walked and came up with 10 m.

Let's now go outside the bus - a bystander watched the walker and observed that by the time he finished his walk, he had covered a distance of 100 m as measured from the traffic light to the bus stop where the bus stopped to let him off.

How can the same person walk both 10 m and 100 m at the same time?

Reference Frames

That's the concept of a Reference Frame.

The Reference frame is the system in which measurements are taken - and the Reference frame can be stationary, accelerating or moving at a constant velocity.

Your measurement depends on which reference frame you're using.

Reference Frames

In the Bus reference frame the walker covered 10 m. But in a reference frame attached to the earth, he covered 100 m.

And if you use a reference frame attached to the sun, the same person covered a distance of 149,000 m! Why is the last distance so great?

Reference Frames

Because the earth is rotating about the sun at a tangential velocity of 29,900 m/s. So in 5 s, if you were sitting on the sun, you would see the bus rider covering a distance of 29,900 m/s * 5 s = 149,000 m?

Can you think of other reference systems that would give you a different measured difference? Remember - just like the earth is moving about the sun, the sun is also moving.

Star System
Nebula
Galaxy
Cluster
Supercluster

Inertial Reference Frames

There is a special kind of reference frame - it is called an Inertial Reference frame and it is special in that Newton's First Law of Motion is valid. In other words, an object in such a frame will only accelerate if a net non-zero external force is applied.

It is a frame that is moving at a constant velocity which includes a frame at rest (since 0 m/s is a constant velocity).

Accelerating reference frames are not inertial reference frames. Reference frames attached to a bus that is increasing its speed or to a merry go round are not inertial reference frames. Why is this?

Inertial Reference Frames

Take the case of the bus. If a golf ball is in the aisle at the front of the bus and the bus accelerates from a stop sign, what would the golf ball do? You'd see it move backwards down the aisle. Without anybody applying a force to it!

If you're on a merry go round and you throw a ball across the merry go round to your friend - she sees the ball curve away - without any force pushing it to the side.

Clearly, Newton's First Law isn't working here.

10 Which of the following is an inertial reference frame?

○ A An airplane increasing its speed during takeoff.

○ B A racing car maintaining a constant speed while going around a curve.

○ C A racing car decreasing its speed after it crosses the finish line.

○ D An airplane flying with a constant speed and direction.

○ I need help

Answer

https://www.njctl.org/video/?v=TLHovPX98NQ

11 Multi-Correct: What conditions exist in an inertial reference frame?

- ☐ Newton's First Law is valid.
- ☐ The frame is moving at a constant velocity.
- ☐ The frame is increasing its velocity.
- ☐ The frame is decreasing its velocity.
- ☐ E I need help

Answer

https://www.njctl.org/video/?v=W2iY2f01B7w

Mass and Weight

Mass is the measure of the inertia of an object; the resistance of an object to acceleration by an external net non-zero force.

Weight is the force exerted on that object by gravity. Close to the surface of the Earth, where the gravitational force is nearly constant, the weight is defined as the magnitude of this force:

$$W = \left|\vec{F}_G\right| = \left|\frac{GM_E m}{r^2}\right| = \left|m\vec{g}\right|$$

g is the gravitational acceleration due to the Earth attracting the object and is equal to 9.8 m/s^2.

https://www.njctl.org/video/?v=D2c1q_-auUU

Mass and Weight

Mass is measured in kilograms and weight is measured in Newtons because it is a force.

$$W = \left|\vec{F_G}\right| = \left|\frac{GMm}{r^2}\right| = \left|m\vec{g}\right|$$

Does the value of the mass or the weight of an object change depending on where it is?

Mass and Weight

Mass is a constant, no matter where the object is located.

$$W = \left|\vec{F}_G\right| = \left|\frac{GMm}{r^2}\right| = \left|m\vec{g}\right|$$

However, the weight of an object can change depending on what planet it is on.

The above equation (Newton's Law of Universal Gravitation) shows the dependence of the weight on the mass and radius of the planet.

12 What is the weight of a 32.3 kg object on the earth? Use g = 10.0 m/s^2.

○ 323 N

○ 404 N

○ 479 N

○ 512 N

○ I need help

https://www.njctl.org/video/?v=Y9X61c1Vi4o

Answer

13 What is the weight of a 32.3 kg object on the moon? Use $g_{moon} = 1.67\ m/s^2$.

○ 53.9 N

○ 80.3 N

○ 88.6 N

○ 95.8 N

○ I need help

Answer

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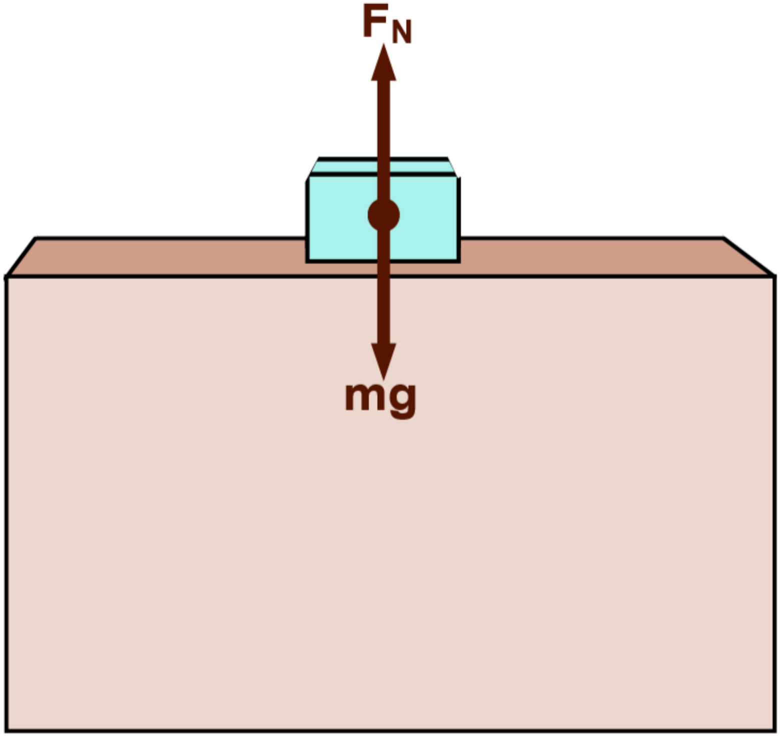
Normal Force and Weight

The Normal Force, F_N, is always perpendicular to the surface that is creating it.

Weight, mg, is always directed downward.



We know where the weight force comes from - but what is the origin of the Normal Force?

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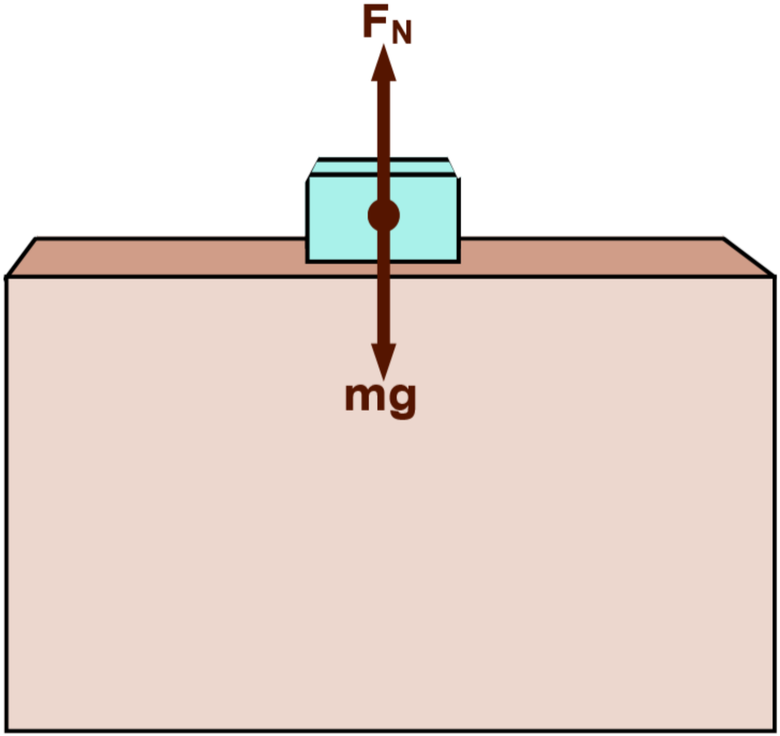
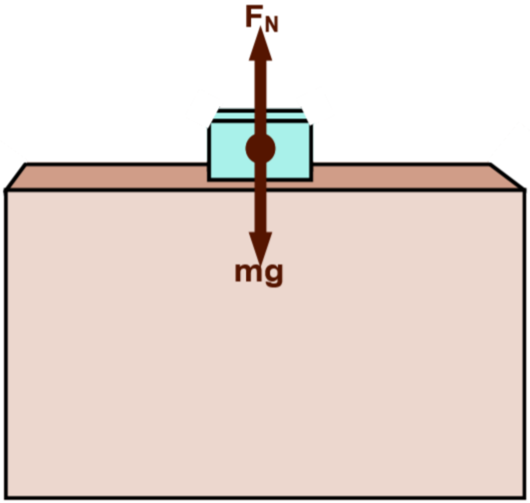
Normal Force and Weight

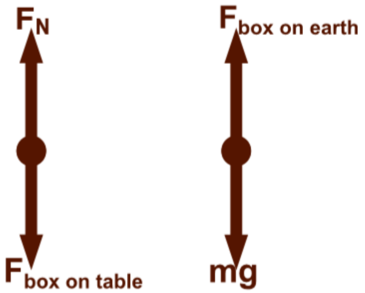


The Normal Force is a consequence of Newton's Third Law and is due to the electrons in the table repelling the electrons in the box - which results in an upward, Normal Force.

The box is being pulled down by gravity (mg), and the Normal Force is pushing up on the box. Is this a Newton's Third Law action-reaction pair of forces?

Normal Force and Weight

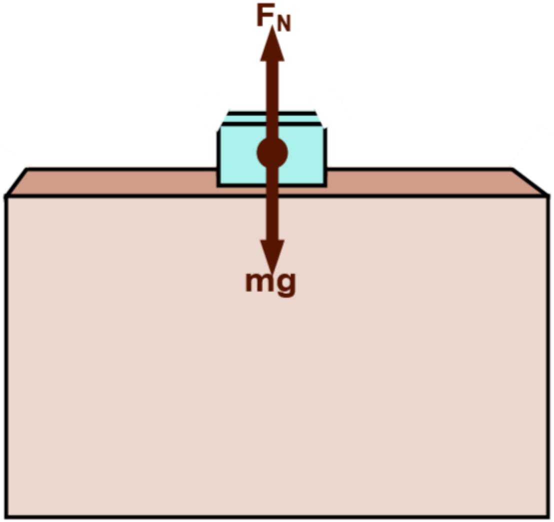




No! The Normal force and the gravitational force, mg, both act on the box. Action reaction force pairs act on *different* objects. The Normal force and the force that the box exerts on the table is an action reaction pair.

The force that the earths' gravity (mg) exerts on the box is an action reaction pair with the gravitational force that the box exerts on the earth.

Normal Force and Weight

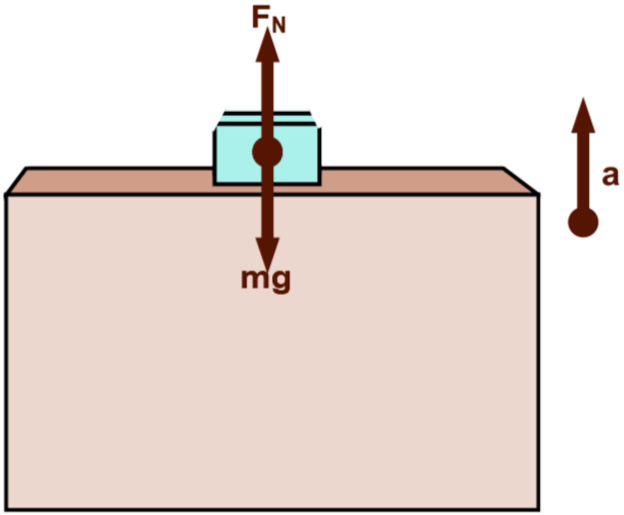


If the table is not accelerating in the y direction and the box is not moving up and down on the table, then $F_N = mg$.

$$\Sigma F_y = F_N - mg = ma_y = 0$$

$$F_N = mg$$

Normal Force and Weight



But, if the table is in an elevator and is accelerating upwards, then we have:

$$\Sigma F_y = F_N - mg = ma_y$$

$$F_N = mg + ma_y$$

The Normal Force is greater than the weight. If the box was replaced with a person, the person would feel heavier than their typical weight.

Thus we have another name for the Normal Force - it is also called the Apparent Weight.

14 A 42.3 kg object rests on a table. What is the Normal force exerted by the table on the object? Use $g = 10.0\ m/s^2$.

○ 393 N

○ 423 N

○ 516 N

○ 603 N

○ I need help

https://www.njctl.org/video/?v=-LYYAm7gZ6g

Answer

15 A 42.3 kg object rests on a table. The table is placed in an elevator and accelerates upwards at 1.55 m/s^2. What is the Normal force (Apparent Weight) exerted by the table on the object?

Use $g = 10.0$ m/s^2.

○ 339 N

○ 404 N

○ 489 N

○ 553 N

○ I need help

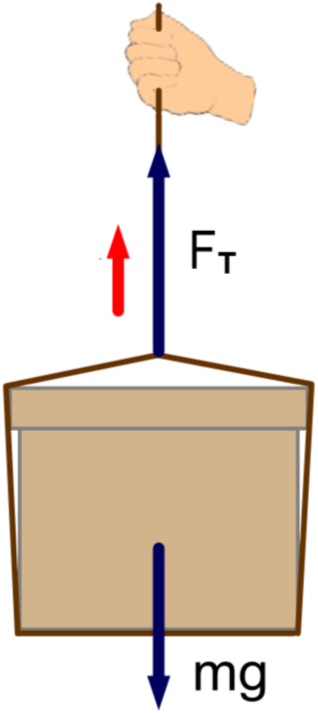
https://www.njctl.org/video/?v=yfhLJYZA7Gw

Answer

Tension Force

When a cord or rope pulls on an object, it is said to be under tension, and the force it exerts on the object is called a tension force, F_T.

Are the forces shown on the diagram to the right an action reaction pair?



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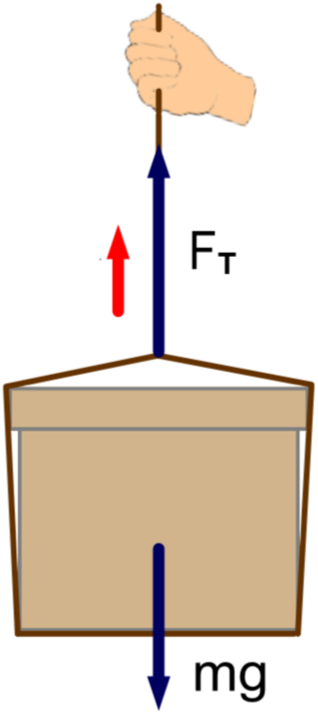
Tension Force

No, they are not an action reaction pair. The Tension force and mg are both operating on the box. Action reaction pairs operate on different objects.

If the hand is pulling the box up with a constant velocity, what is the relationship between F_T and mg?

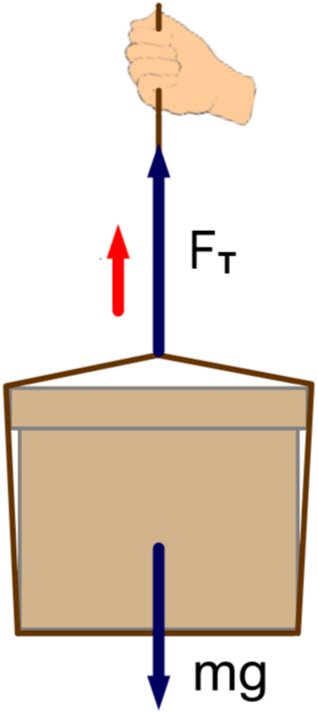


Tension Force

They are equal. If the pail is moving with a constant velocity, then $a_y = 0$.

$$\Sigma F_y = F_T - mg = ma_y = 0$$

$$F_T = mg$$



16 A rope affixed to the ceiling is holding a bucket of water of mass 22.4 kg. What is the Tension force in the rope? Use $g = 10.0\ m/s^2$.

○ A 44.8 N

○ B 448 N

○ C 224 N

○ D 22.4 N

○ E I need help

https://www.njctl.org/video/?v=5OMfXhxpJvQ

Answer

17 A rope is tied to a bucket of water of mass 22.4 kg. The bucket is pulled upwards with an acceleration of 2.77 m/s^2. What is the Tension force in the rope?

Use g = 10.0 m/s^2.

○ 228 N

○ 255 N

○ 286 N

○ 304 N

○ I need help

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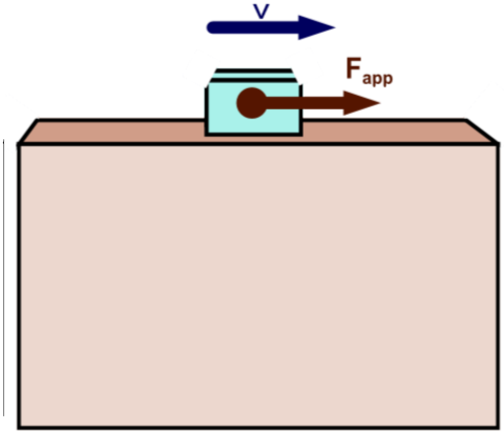
Answer

Friction

When we first discussed Newton's First Law, the concept of friction as a force that opposes motion was introduced without any mathematical details, or even qualitative discussion.



It's now time to do that. Look at the box to the right. You're pulling the box with an applied Force, F_{app}.

Is it harder to get the box moving or keep it moving?

What happens if you increase the mass of the box?

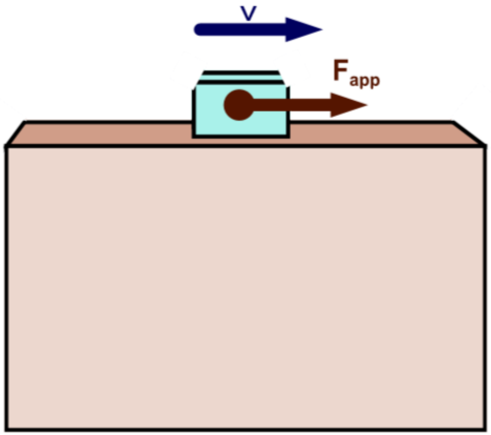
https://www.njctl.org/video/?v=Xhu6fK9ngKI

What happens if you pull the box over a surface of ice, compared to a surface of sandpaper?

Demo

Friction

You probably found it harder to get the box moving - once moving, it needed less F_{app} to keep it moving.



The more massive the box - the more F_{app} was required to start it moving and to keep it moving.

And, it is much easier to pull a box over an icy surface versus over sandpaper - a smaller F_{app} is needed.

So, the friction force appears to depend on whether the object was at rest or moving, its mass and the surface it was moving on.

Friction

We'll first address the issue of the difference in F_{app} required to overcome the friction of a stationary object and a moving object by distinguishing between two types of friction - static and kinetic.

Static friction force is the force that works to prevent the motion of a stationary object.

Kinetic friction force is the force that acts opposite to the motion of a moving object.

Friction

In both Static and Kinetic friction, it is harder to move a more massive object - so there is a dependence on the Normal force - the force that the surface is exerting on the stationary or moving object.

Also - both types of friction depend on the type of material that the object and the surface are made of.

This is represented by the coefficient of static friction (μ_s) and the coefficient of kinetic friction (μ_k). These coefficients have been measured for many material interfaces.

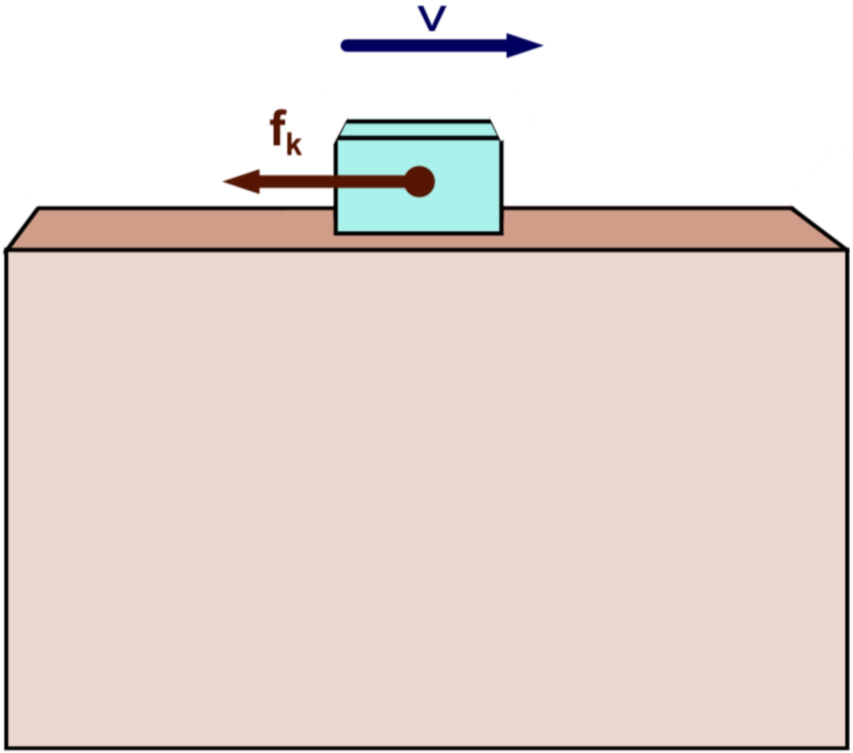
It is interesting to note that the contact area between the object and surface does not affect the friction force.

Kinetic Friction

Friction forces are always parallel to the surface exerting them.

Kinetic friction is always directed opposite to the direction that the object is moving and has magnitude:

$$f_k = \mu_k F_N$$



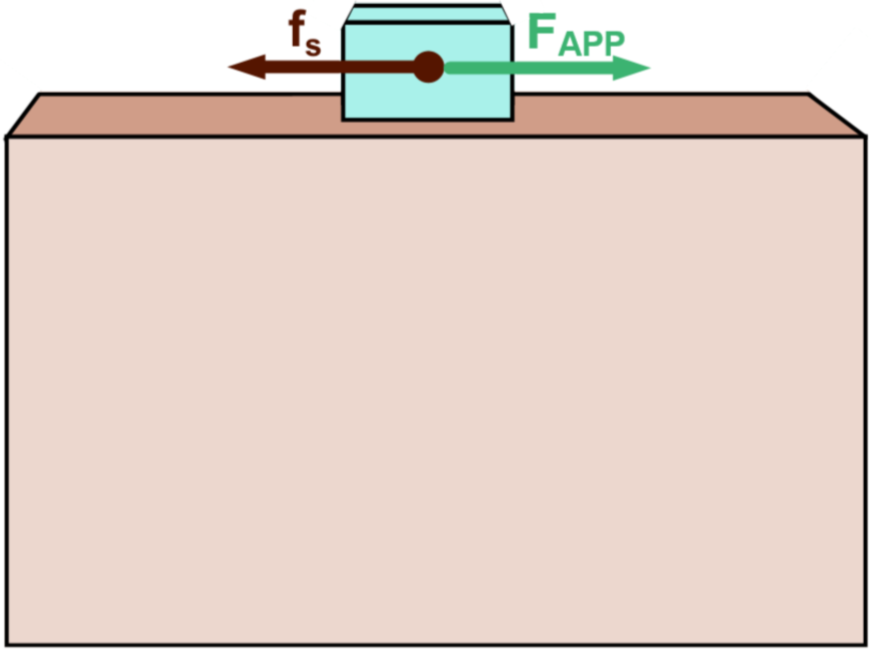
Static Friction

Static friction is less than or equal to a maximum possible value (given by the equation below), and is opposite to the direction of the external net applied force.

The static friction force seeks to maintain the original relative motion between the two objects.

Its magnitude is:

$$f_s \leq \mu_s F_N$$



What is the significance of the inequality?

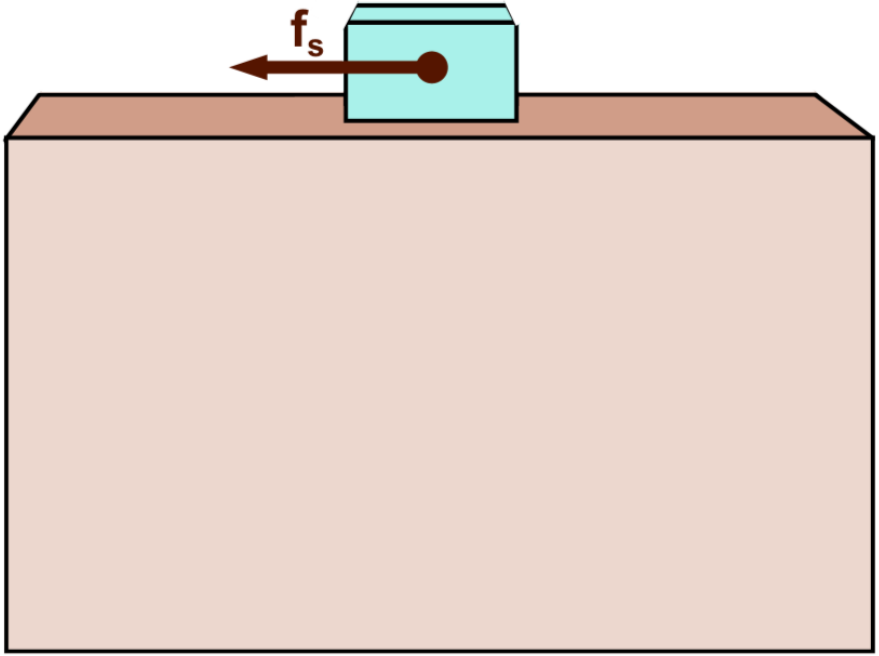
Static Friction

If there were no inequality, then there would always be a static friction force acting on the object.

What would that look like?

An object could never be at rest! There would always be a friction force trying to accelerate it.

$$f_s \leq \mu_s F_N$$



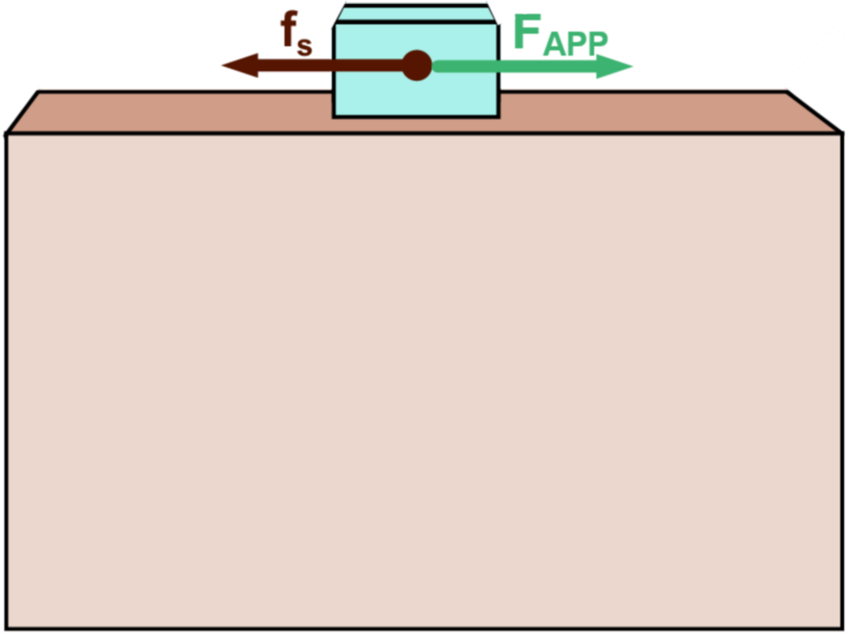
Static Friction

However, when an object is not subject to an external force, there will be no friction force acting on it.

As an external force is applied, the static friction force will increase to a maximum value of $\mu_s F_N$.

At this point, the object will move, and the kinetic friction force will take over.

$$f_s \leq \mu_s F_N$$

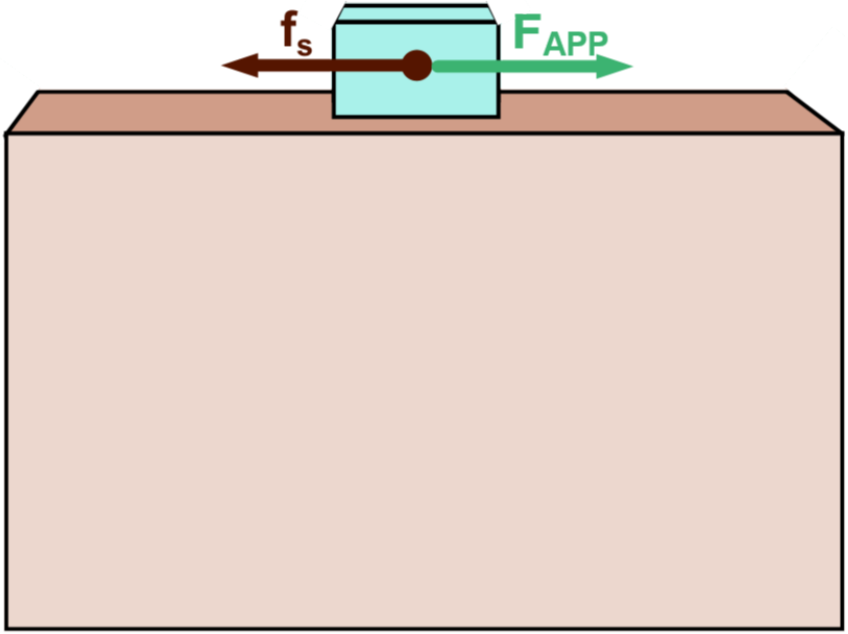


Static Friction

The maximum static friction force is greater than the kinetic friction force.

Another way to say this is $\mu_s \geq \mu_k$.

Knowing this, what happens at the point when the applied force is greater than the maximum static friction force?

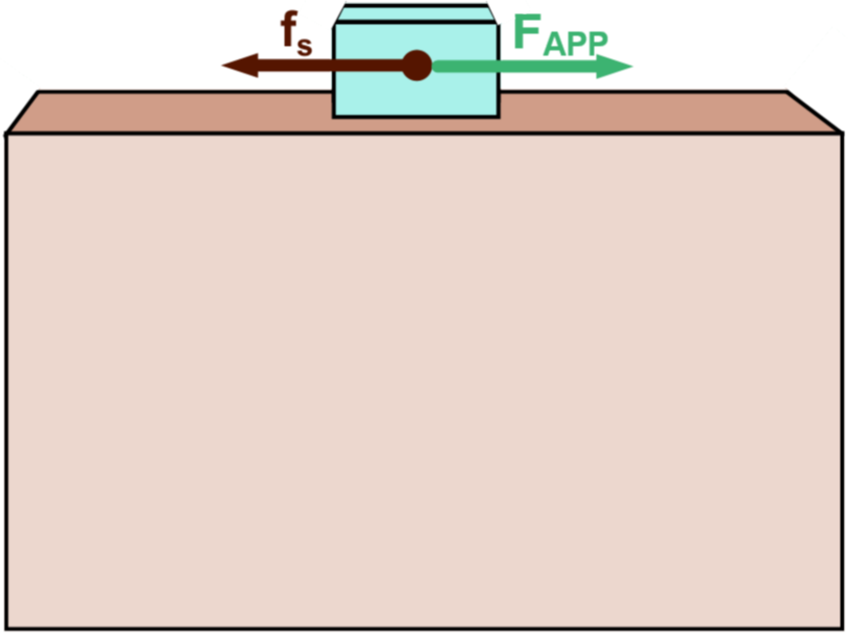


You've probably experienced this if you've ever moved a couch across a room........

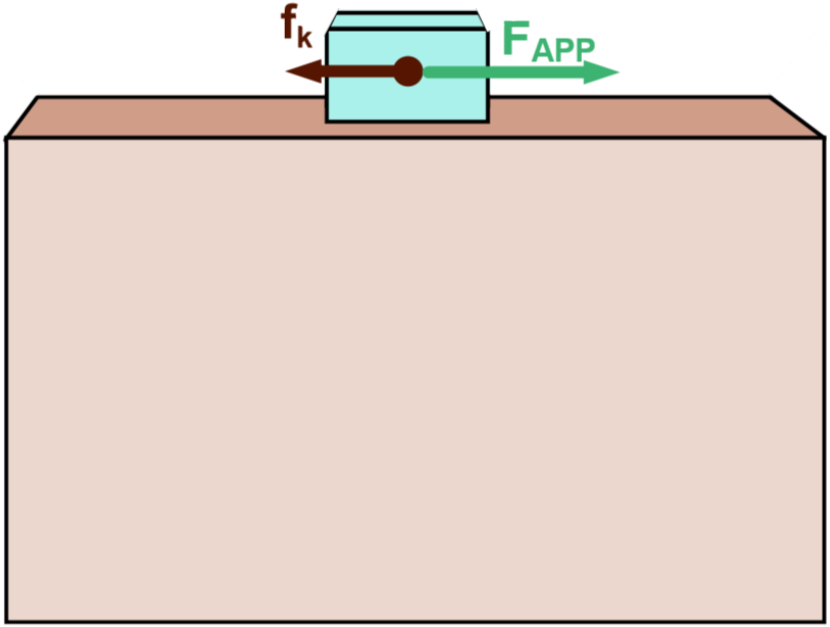
Static Friction

Assume you start pushing with a small force - and nothing happens because the static friction force is increasing to match your applied force. ΣF=0, so no acceleration or movement.

Right after the applied force is greater than the maximum static friction force, the object moves, and the smaller kinetic friction force is opposing your applied force.



Static Friction



The opposing friction force decreased rapidly - and you're still pushing with the same applied force.

The couch accelerates - you're not ready for it - and you might fall over.

Note how f_k is less than the maximum f_s.

18 Multi-Correct: How is the kinetic friction force different from the static friction force?

☐ The coefficent of kinetic friction is greater than the coefficient of static friction.

☐ The coefficent of kinetic friction is less than the coefficient of static friction.

☐ Kinetic Friction is constant. Static Friction depends on the applied force.

☐ Kinetic Friction is in the same direction of movement, static friction is opposite.

☐E I need help

https://www.njctl.org/video/?v=z5EuOYi-HFs

Answer

19 What is the kinetic friction force on an object of mass 44 kg as it moves over a rough surface where $\mu_k = 0.75$? Use $g = 10\ m/s^2$.

○ A 330 N

○ B 390 N

○ C 415 N

○ D 440 N

○ E I need help

https://www.njctl.org/video/?v=frp_4vGhTVI

Answer

20 What is the maximum static friction force on an object of mass 44 kg on a rough surface where $\mu_s = 0.87$?

Use $g = 10\ m/s^2$.

○ 320 N

○ 380 N

○ 440 N

○ 470 N

○ I need help

https://www.njctl.org/video/?v=EGRfyl1IHuc

Answer

Explain what happens to the friction force on a object when the applied force is increased from zero to an amount greater than the maximum static frictional force.

https://www.njctl.org/video/?v=v6C9FqTocqI

Answer

Free Body Diagrams

A free body diagram is a drawing that is used in order to show all the forces acting on an object. Drawing free body diagrams can help when trying to solve for unknown forces or determining the acceleration of the object.

Click here for a *Veritasium* video on free body diagrams and the Normal Force!

https://www.njctl.org/video/?v=OXIX01pymq0

Free Body Diagrams

You don't have to be an artist to draw free body diagrams as you'll see shortly. But first, let's analyze the picture from the preceding page. Here it is again.

What forces are acting on the big question mark box?

Free Body Diagrams

The simulated person is pushing the object to the right.

Gravity is pulling the box down.

The surface (which is not shown, but is implied!) is pushing up on the box with the Normal force.

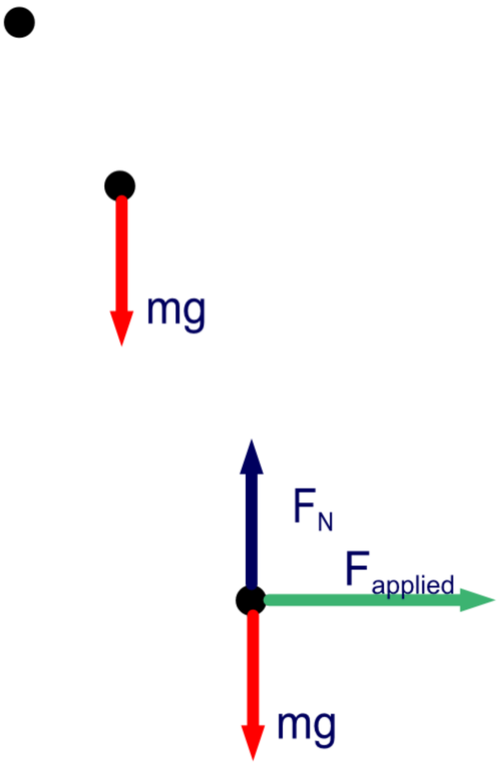
There are three forces. We're now ready to translate this sketch into a simplified form with vectors.

Free Body Diagrams

1. Draw and label a dot to represent the box. See, you don't even have to be able to draw a stick figure to do free body diagrams.

2. Draw an arrow from the dot pointing in the direction of one of the forces that is acting on that object. Label that arrow with the name of the force.

3. Repeat for every force that is acting on the object. Try to draw each of the arrows to roughly the same scale, bigger forces getting bigger arrows.

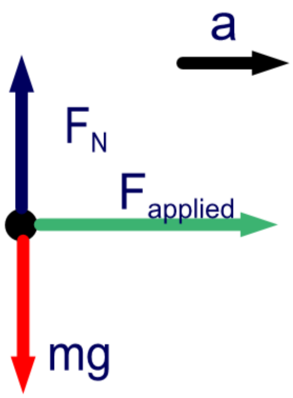


Free Body Diagrams

4. Once you have finished your free body diagram, recheck it to make sure that you have drawn and labeled an arrow for every force. This is no time to forget a force.

5. Draw a separate arrow next to your free body diagram indicating the likely direction of the acceleration of the object. This will help you use your free body diagram effectively.

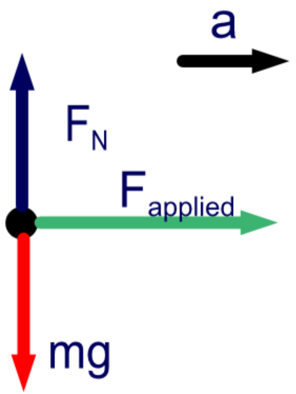
6. Repeat this process for every object in your sketch.



Free Body Diagrams

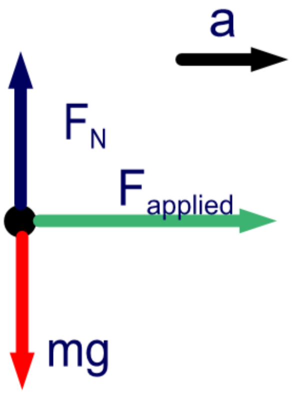
Don't worry if the force vectors aren't the right size - they're just there to give you an idea of which way the object will accelerate - the mathematics will work out and give you the correct answer.

And, if you choose the wrong direction for the acceleration, don't worry about that either - after you do the math, the acceleration will be negative - indicating it's in the opposite direction from what you chose.



Problem Solving

Once the free body diagrams are drawn for each object in the problem, Newton's Second and Third Laws are applied to find the acceleration of the object.



After the acceleration is found, the Kinematics equations are used to find out further information about the displacement and velocity of the objects at future times.

Remember - the acceleration found does NOT tell you which way the object is moving - it only tells you how the velocity is changing!

21 What are the key components of a free body diagram?

○ A All of the forces on the object, along with their direction.

○ B The direction of the expected acceleration.

○ C Force vectors drawn with their approximate magnitude.

○ D All of the above.

○ E I need help

Answer

https://www.njctl.org/video/?v=EoBByulERmU

Given all the forces on an object, describe how you would draw a free body diagram and solve for the motion of an object.

https://www.njctl.org/video/?v=9BLN0ZsXrmA

Answer

You are pushing a wagon on a sidewalk with a kinetic friction force opposing your force. Draw and label a complete free body diagram, with the expected acceleration of the object.

https://www.njctl.org/video/?v=wfgDyQeM8ok

Answer

Resolving Forces into Two Dimensions

https://www.njctl.org/video/?v=s5syzJ5UfaQ

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Resolving Forces

So far, in Dynamics problems, you've only had to deal with motion in one dimension - all the forces were in the same dimension, typically either the x or y axis.

Yet, there is a hint of two dimensional motion in problems you've already solved.

Can you figure out where you used forces in two dimensions in solving a problem?

Resolving Forces

Friction problems.

If an object was moving along the x axis with a friction force opposing its motion, the friction force was calculated by using the Normal force in the y direction.

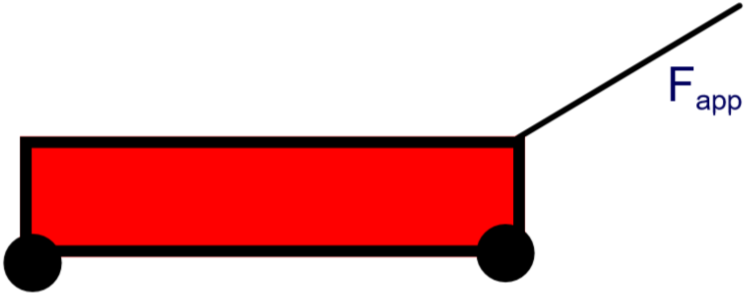
By multiplying the Normal force by the coefficient of friction, the friction force along the x axis was calculated.

But, these were two separate and independent calculations.

Resolving Forces

The various forces on an object don't always act in one dimension.

An easy example is when you're pulling a wagon at an angle to the horizontal as seen below.




Direction of Motion

Resolving Forces

In order to solve problems using forces acting at an angle, like the wagon on the previous slide, we must find the horizontal (x) and vertical(y) components of the forces.

The original force vector is now represented as the vector sum of its components. These components will show the motion of the object in the x and y direction *independently* - and when added, as vectors, will match the actual motion.

Most of the time, it's easier to resolve a force into two perpendicular vectors on the x and y axis. Later on, there will be cases when the x and y axis will be rotated - but we won't worry about that now.

Resolving Forces

Here's the Etch a Sketch again - the knobs move the metal pointer either along the rod in the x direction or the y direction.

The combined motion can yield a quite complex figure - but moving the x knob only affects motion in the x direction - and the y knob only affects motion in the y direction.

If the motion of an object in the x direction and y direction is known, how can the total motion of the object be found?

Answer

https://www.njctl.org/video/?v=F_jg9dYBW6I

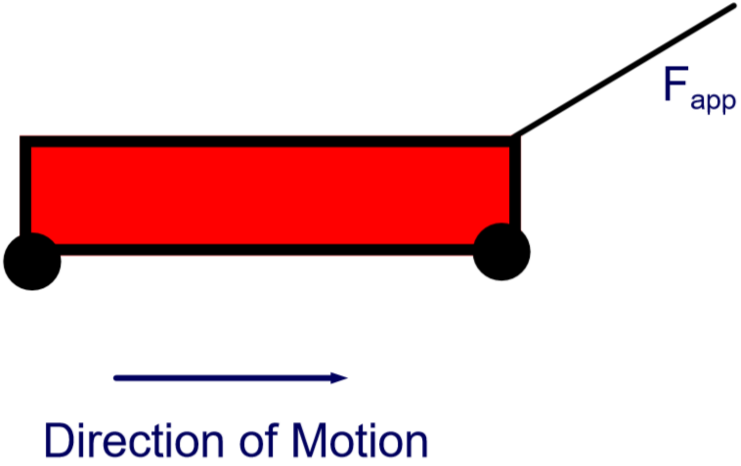
An object is moving in the positive x direction on a surface with friction. What equations are used to translate the y direction forces into a friction force acting in the negative x direction?

https://www.njctl.org/video/?v=7EC77imX2O4

Answer

Resolving Forces

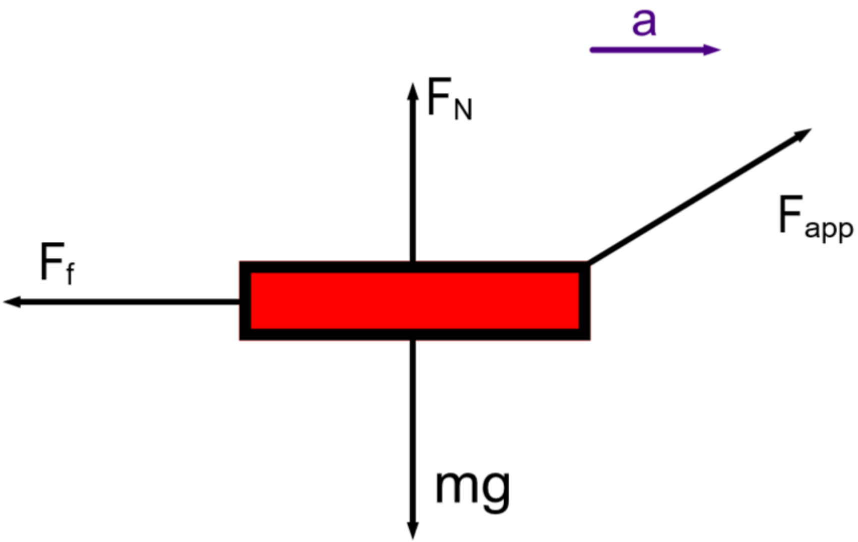
Consider the wagon being pulled down the street.The wagon has a handle that is not vertical or horizontal, but at an angle to the horizontal. This means the wagon is being pulled up and to the right at the same time.



https://www.njctl.org/video/?v=8vWt6yJFLQo

Resolving Forces

A free body diagram would include this, and all other forces, as seen below.



Yes, a real free body diagram would just have a dot for the wagon - but the wagon body is shown to help you follow the forces.

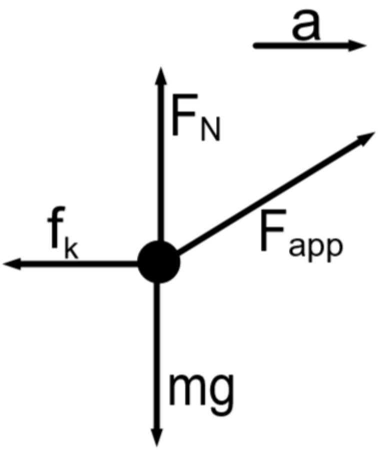
Resolving Forces

F_N and mg act in the y direction and can be handled together. f_k acts only in the x direction.

But, F_{app} is a problem. It acts in both the x and y dimensions.

We need to separate F_{app} into its components along the x and y axis.

What mathematical discipline will accomplish this separation?

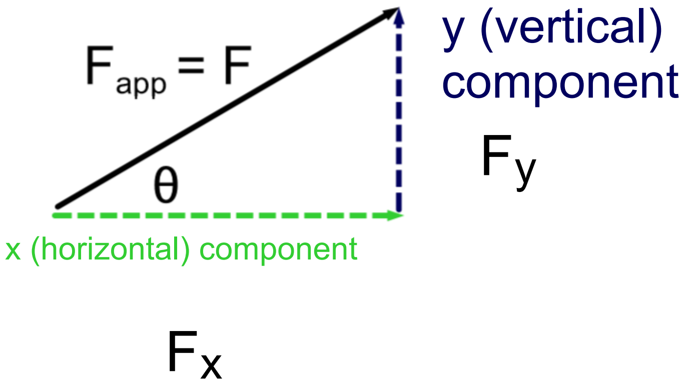


Resolving Forces

Trigonometry.

To simplify the notation, let $F_{app} = F$. The diagram to the right shows how we can express F as the vector sum of F_x and F_y.

Another question - given that we know F and θ, what trigonometric functions can we use to find F_x and F_y?



$F_{app} = F$

θ

y (vertical) component

F_y

x (horizontal) component

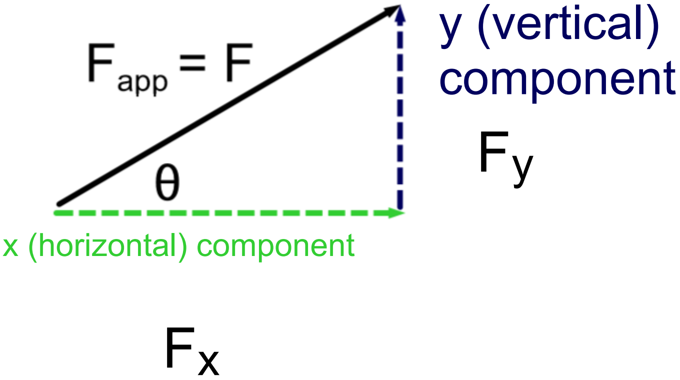
F_x

Resolving Forces

Cosine function.

Sine function.

Let's try an example.



$F_{app} = F$

θ

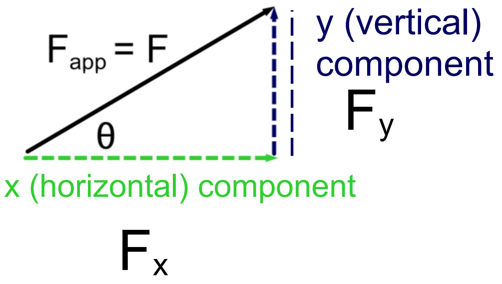
y (vertical) component

F_y

x (horizontal) component

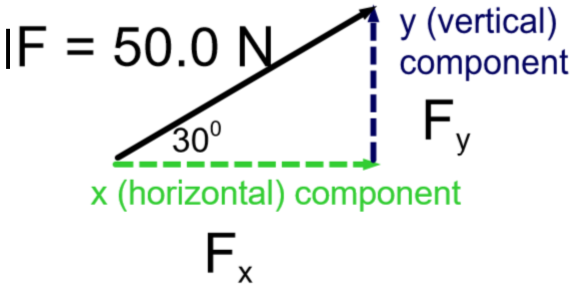
F_x

Resolving Forces

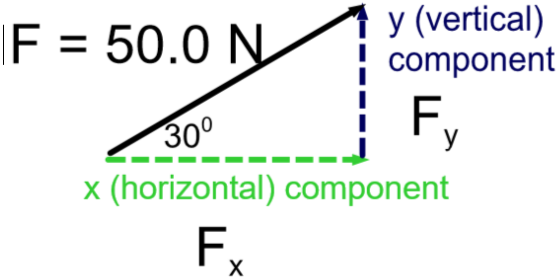


F needs to be expressed in component form - with one component on the x axis and the other on the y axis.

Then, they can be lined up in single dimensions with the other forces. Let's work an example when the pulling force is 50.0 N at 30^0 with respect to the ground.



Resolving Forces



F = 50.0 N

y (vertical) component

F_y

30^0

x (horizontal) component

F_x

Use the cosine function where cos θ = adjacent /hypotenuse to find F_x

$$\cos\theta = {}^{adj}\!/_{hyp}$$

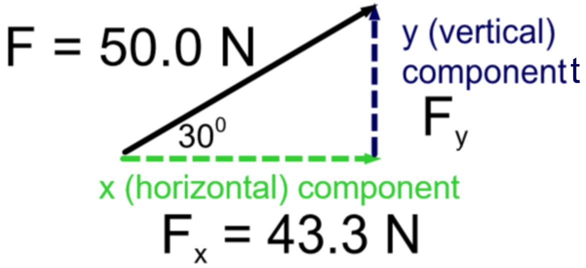
$$\cos\theta = \frac{F_x}{F}$$

$$F_x = F\cos\theta$$

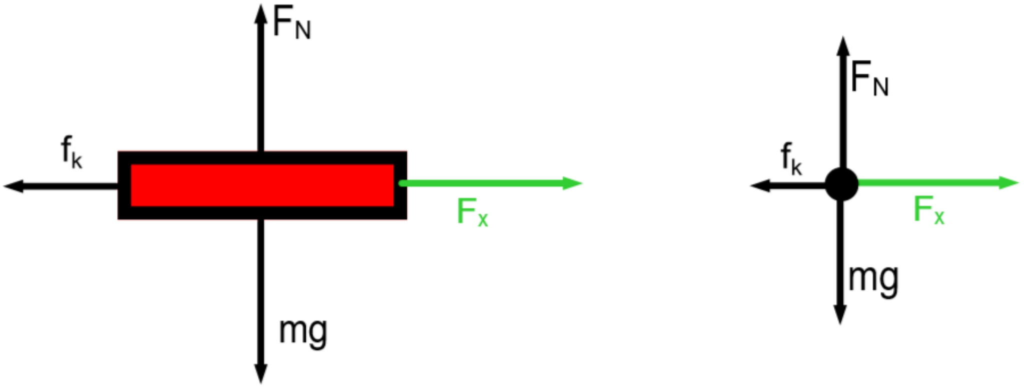
$$F_x = 50.0\cos 30^0$$

$$F_x = 43.3N$$

Resolving Forces

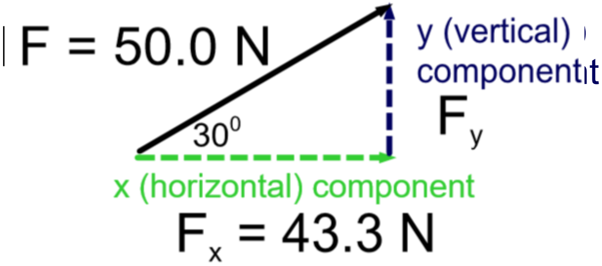


The horizontal (x) component of the force is equal to 43.3 N. This is now added to the free body diagram:



But the vertical (y) component of the original force is temporarily lost... it must be found next.

Resolving Forces



Use the sine function where sin θ = opposite /hypotenuse to find F_y

$$\sin\theta = {}^{opp}\!/_{hyp}$$

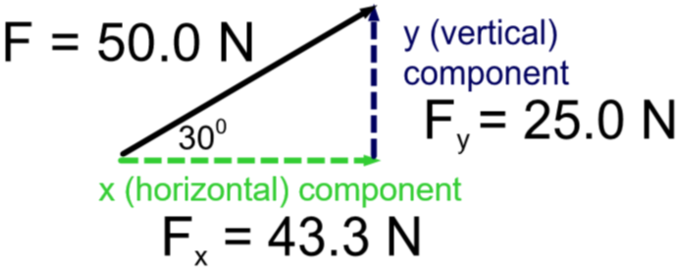
$$\sin\theta = \frac{F_y}{F}$$

$$F_y = F\sin\theta$$

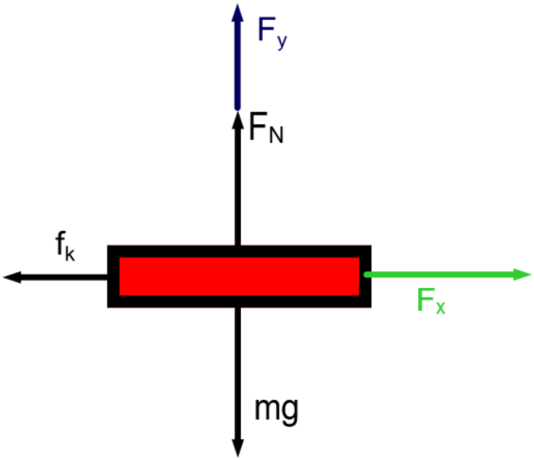
$$F_y = 50.0\sin 30^0$$

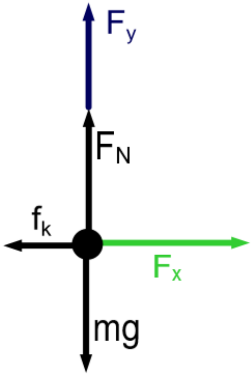
$$F_y = 25.0N$$

Resolving Forces

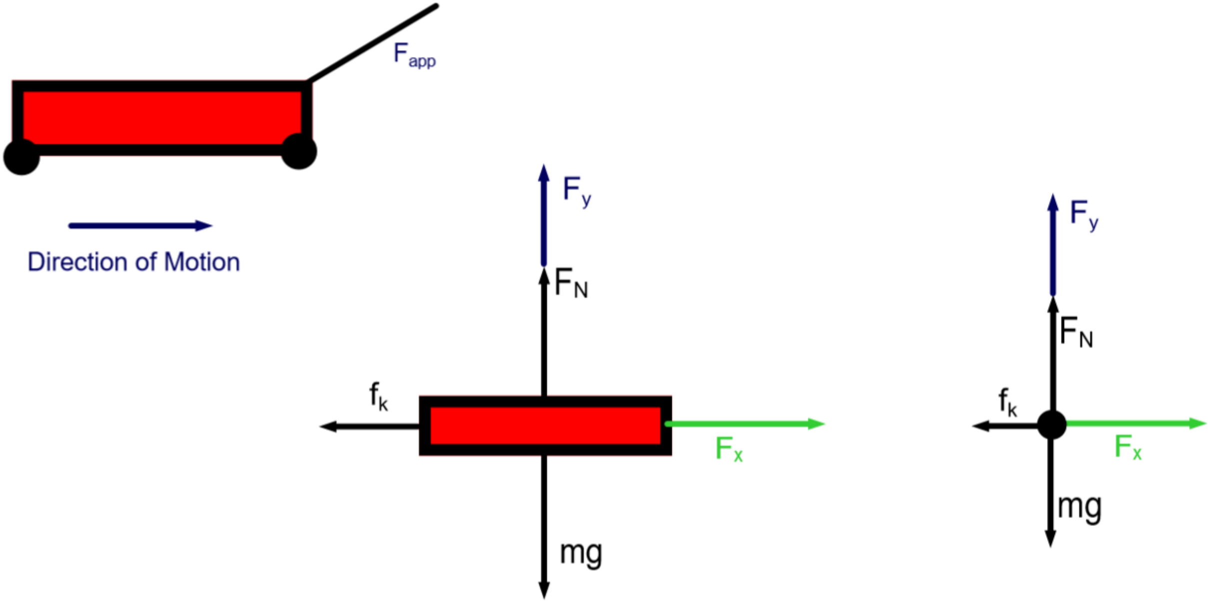


The vertical (y) component of the force is equal to 25.0 N. This is now added to complete the free body diagram:





Resolving Forces



Notice that our original force F_{app} *is no longer shown... it has been replaced by its x and y components!*

22 What are the x and y components of the Force vector shown?

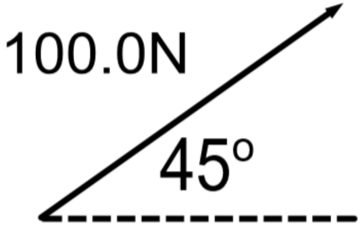
○ A 35 N, 35 N

○ B 71 N, 71 N

○ C 71 N, 35 N

○ D 35 N, 71 N

○ I need help



Answer

https://www.njctl.org/video/?v=UxiVW-txU8c

23 What are the x and y components of the Force vector shown?

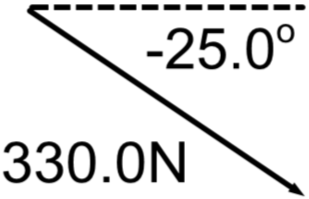
○ A 139 N, -299 N

○ B 139 N, 299 N

○ C 299 N, 139 N

○ D 299 N, -139 N

○ E I nee help



Answer

https://www.njctl.org/video/?v=rtE_gL-H5Lw

24 What are the x and y components of the Force vector shown?

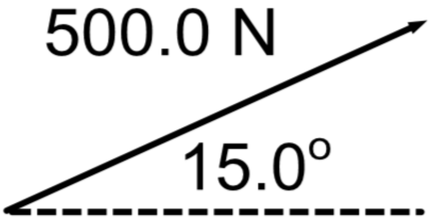
○ A 483 N, 129 N

○ B 129 N, 483 N

○ C 133 N, 133 N

○ D 483 N, -129 N

○ E I need help



Answer

https://www.njctl.org/video/?v=-88fSwPIhik

Two Dimensional Forces

https://www.njctl.org/video/?v=Nvp0K45pVrI

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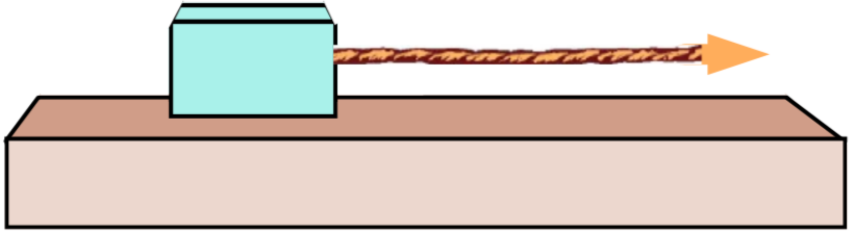
Application of Force Resolution

Now that we can resolve a force at an angle to the x or y axis to its components along the x and y axis, we are ready to solve more complex (and more realistic) motion problems.

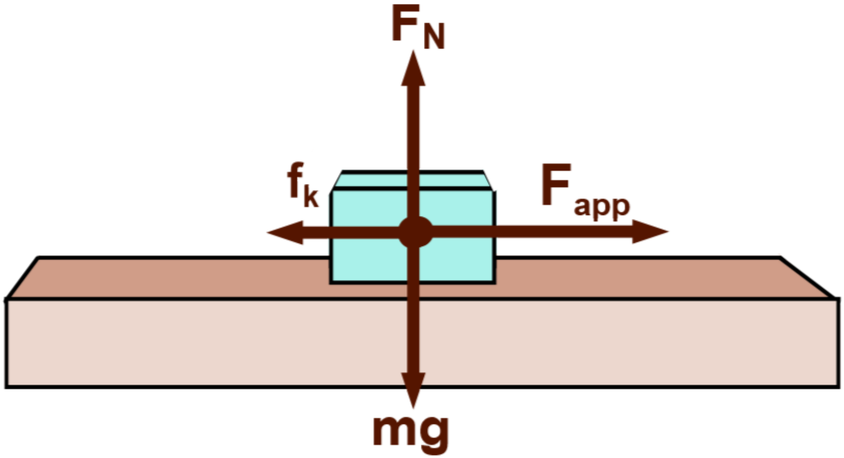
We'll start with a review of determining an object's acceleration in one dimension due to external forces (which are aligned with either the x or y axis).

One Dimensional Applied Force and Friction

For instance, draw the free body diagram of a box being pulled along a rough surface (there is friction between the surface and the box). After this is done, the acceleration along the x axis will be calculated.

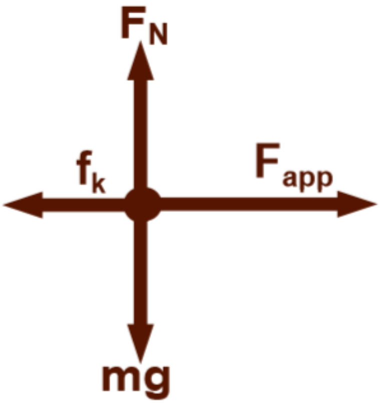


One Dimensional Applied Force and Friction



Now find the acceleration of the box, giventhat the applied force is 20.0 N, the box has a mass of 3.0 kg, and the coefficient of kineticfriction is 0.20.

One Dimensional Applied Force and Friction

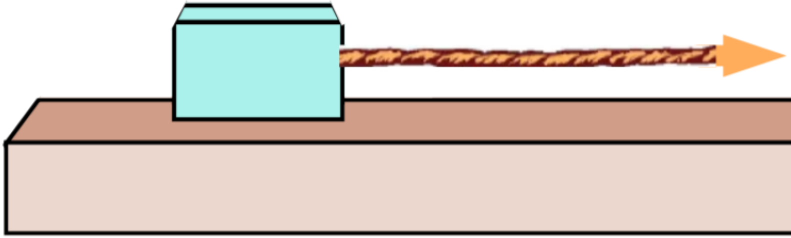


$F_{app} = 20.0\ N$
$m = 3.0\ kg$
$\mu_k = 0.20$

Solve the y axis equation first for F_N so that f_k can be calculated for the x axis equation.

x - axis	**y - axis**
$\Sigma F_x = ma_x$	$\Sigma F_y = ma_y$
$F_{app} - f_k = ma_x$	$F_N - mg = 0$
$F_{app} - \mu_k F_N = ma_x$	$F_N = mg$
$a_x = (F_{app} - \mu_k F_N)/m$	$F_N = (3.0kg)(10m/s^2)$
$a_x = (20N - (0.20)(30N))/3.0kg$	$F_N = 30N$
$a_x = (20N - 6.0N)/3.0kg$	
$a_x = (14N)/3.0kg$	
$a_x = 4.7\ m/s^2$	

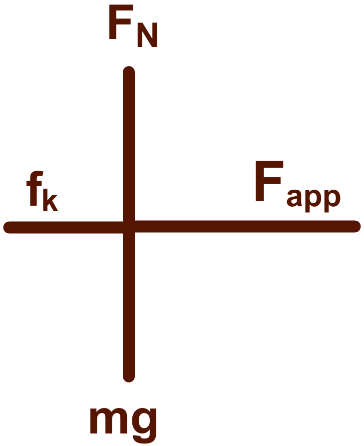
A force of 35 N is applied to a box of mass 4.0 kg on a level surface where μ_k = 0.50. Draw the free body diagram.



https://www.njctl.org/video/?v=rCO0jQUQRh0

Answer

25 Continuing the previous problem, and given the free body diagram and the following values, find the Normal force acting on the box.



$$F_{app} = 35N$$

$$m = 4.0kg$$

$$\mu_k = 0.50$$

$$g = 10\,m/s^2$$

○ 20 N

○ 40 N

○ 60 N

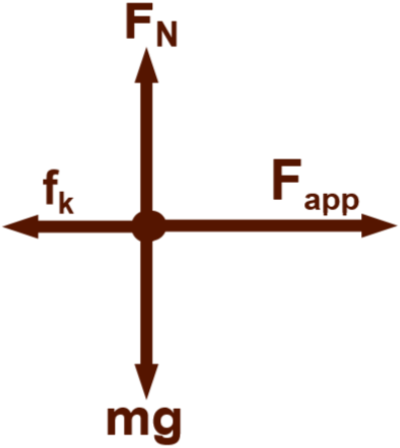
○ 80 N

○ I need help

Answer

https://www.njctl.org/video/?v=uxt8B8mUVz0

26 Continuing the previous problem, and given the free body diagram and the following values, find the acceleration of the box.



$F_{app} = 35N$

$m = 4.0kg$

$\mu_k = 0.50$

$F_N = 40N$

https://www.njctl.org/video/?v=xxoVh7BMFDQ

○ A 1.2 m/s²

○ B 2.3 m/s²

○ C 3.4 m/s²

○ D 3.8 m/s²

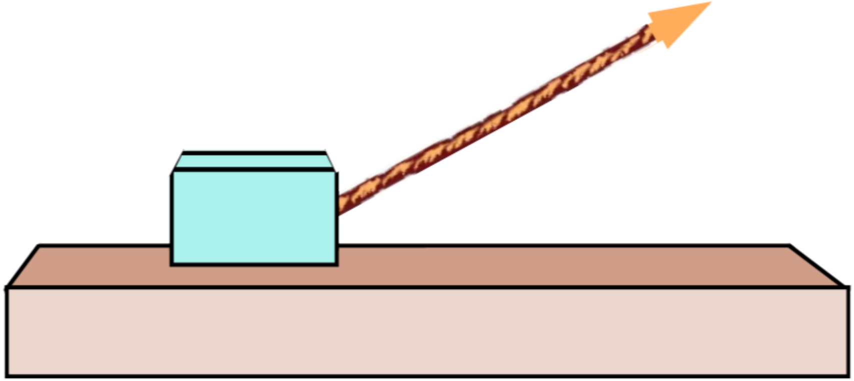
○ E I need help

Answer

Two Dimensional Applied Force and Friction

Now we'll solve problems where the applied force acts at an angle to the friction force, so that they are not parallel or perpendicular with one another.

Take the case of a box being pulled along the floor at an angle.

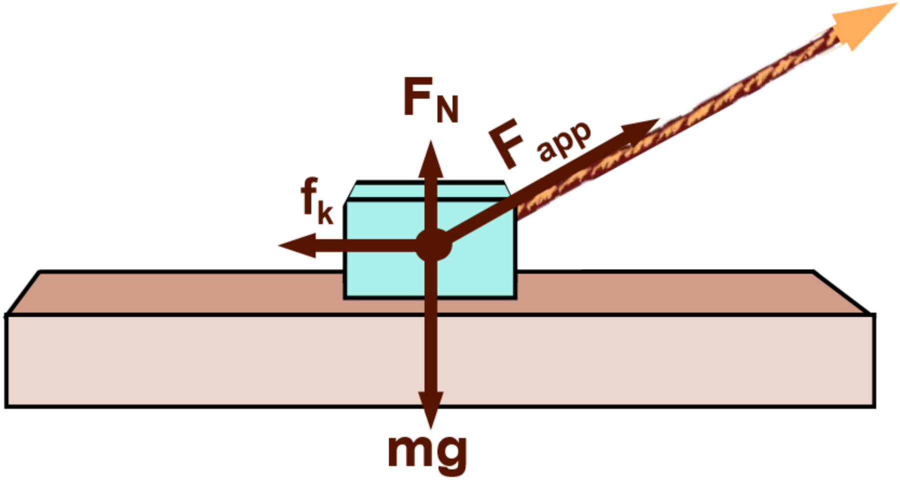


First we do a free body diagram, just as was done previously.

https://www.njctl.org/video/?v=ehcIthwNZrE

Two Dimensional Applied Force and Friction

The next, critical, step is to choose axes. Previously, the x and y axes were used since each axis lined up with the forces...and the accelerations.

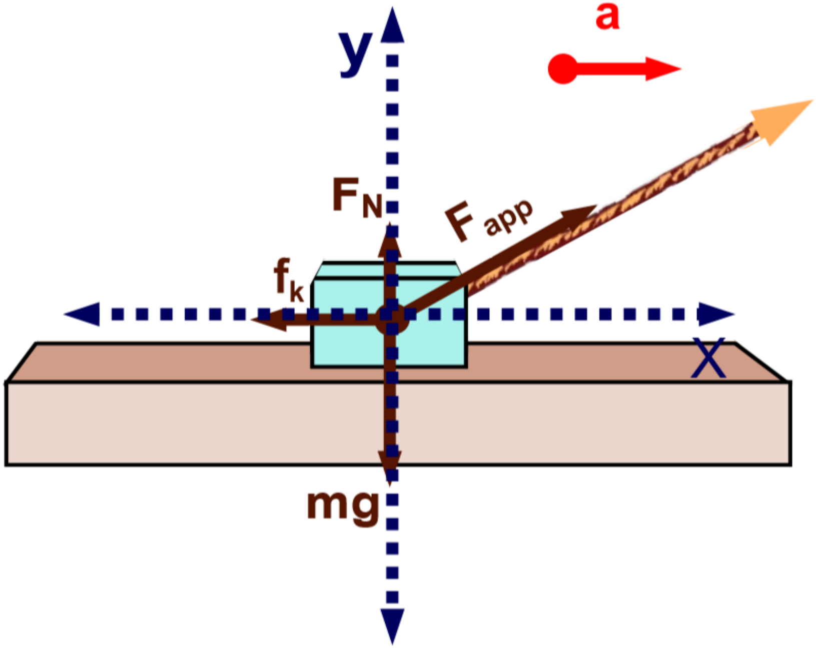


You always have to ask, "In which direction could the object accelerate?" Then make one axis along that direction, and the other one perpendicular.

What's the answer in this case?

Two Dimensional Applied Force and Friction

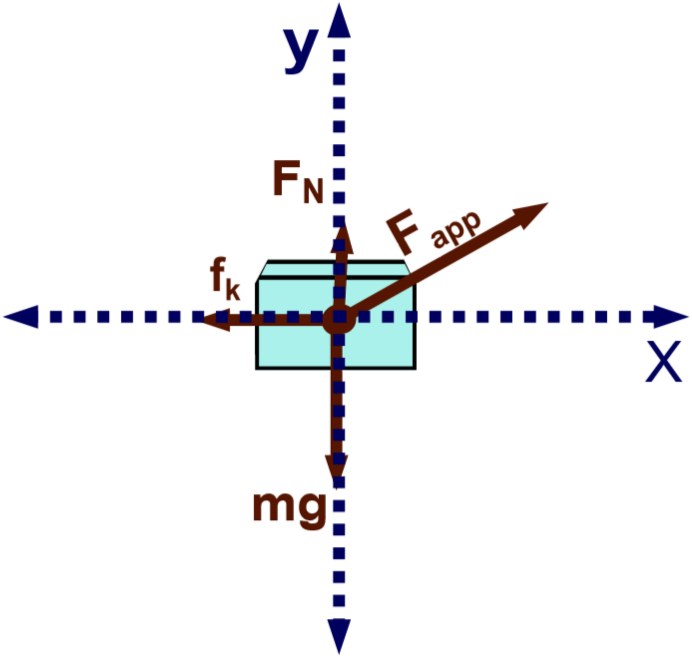
This time, the x and y axes still work since it is assumed that the box will slide along the surface without bouncing up and down ($a_y = 0\ m/s^2$).



Two Dimensional Applied Force and Friction

Now we have to resolve any forces that don't line up with our axes into components that do.

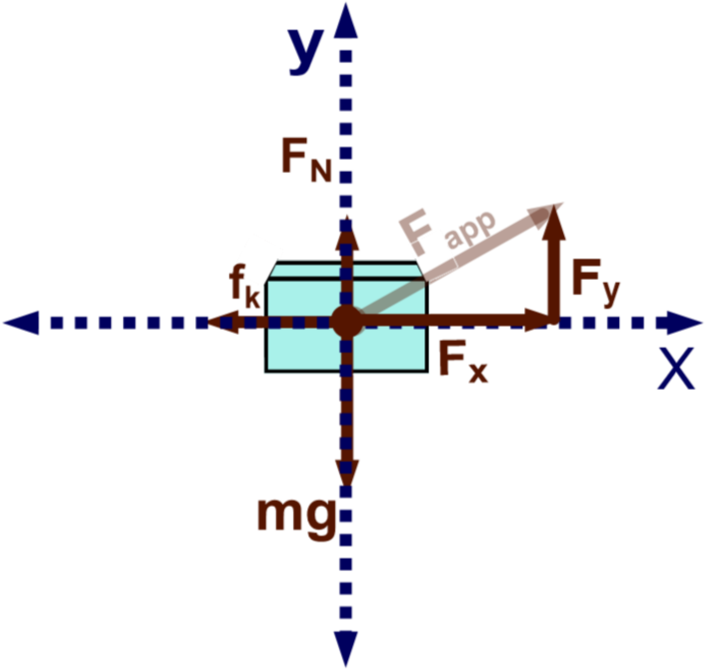
In this case, F_{app} must be resolved into F_x and F_y components. F_N, mg and f_k are good as they are - they are already on the x or y axes.



Two Dimensional Applied Force and Friction

Once that is done,
we can proceed as we did previously, working with the x and y axis components separately.

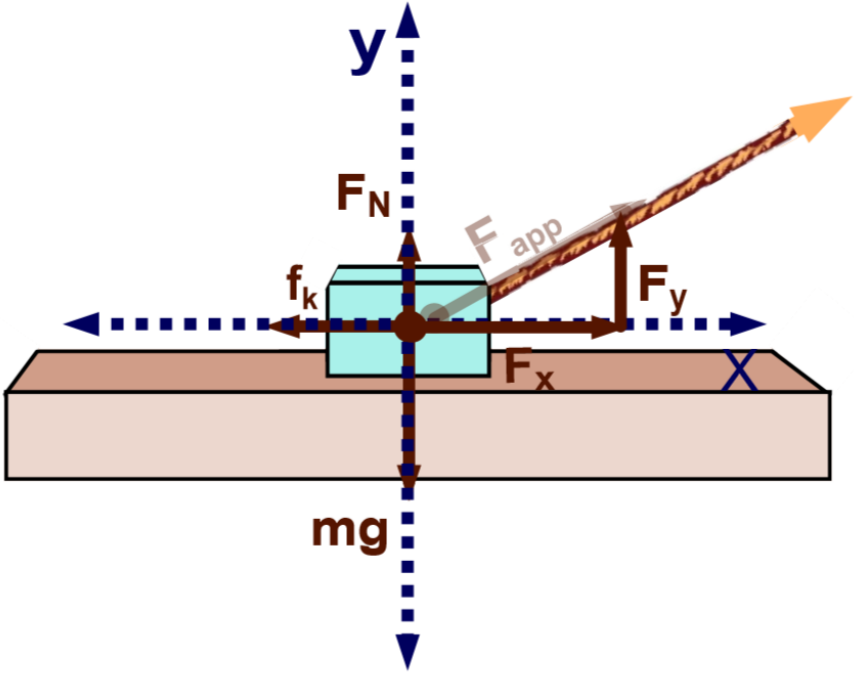
We're now ready to solve a problem.



Two Dimensional Applied Force and Friction

Find the acceleration of a box of mass 3.0 kg, if the applied force is 20.0 N at 37^o above the x axis, and the coefficient of kinetic friction is 0.20.

We solved a similar problem to this one a few slides back - but the applied force was along the x - axis.



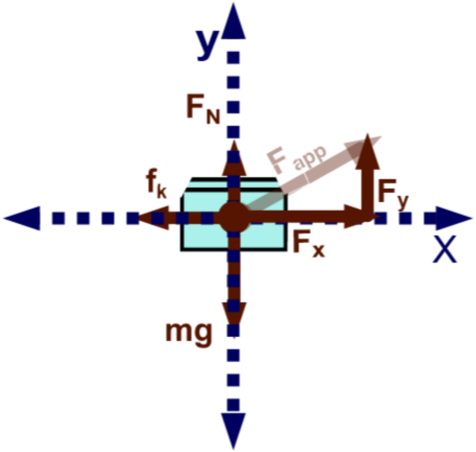
Two Dimensional Applied Force and Friction

$F_{app} = F = 20.0\text{ N at }37^{o}$

$m = 3.0\text{ kg}$

$\mu_k = 0.20$

Remember, solve the y-axis first!



x - axis

$$\Sigma F_x = ma_x$$

$$F_x - f_k = ma_x$$

$$F_x - \mu_k F_N = ma_x$$

$$F\cos\theta - \mu_k mg = ma_x$$

$$a_x = (F\cos\theta - \mu_k F_N)/m$$

$$a_x = (20\text{N} \cos 37^{o} - (0.20)(18\text{N}))/3.0\text{kg}$$

$$a_x = (16\text{N} - 3.6\text{N})/3.0\text{kg}$$

$$a_x = (12.4\text{N})/3.0\text{kg}$$

$$a_x = 4.1\text{ m/s}^2$$

y - axis

$$\Sigma F_y = ma_y = 0$$

$$F_N + F_y - mg = 0$$

$$F_N = mg - F_y$$

$$F_N = mg - F\sin\theta$$

$$F_N = (3.0\text{kg})(10\text{m/s}^2) - (20\text{N})(\sin 37^{o})$$

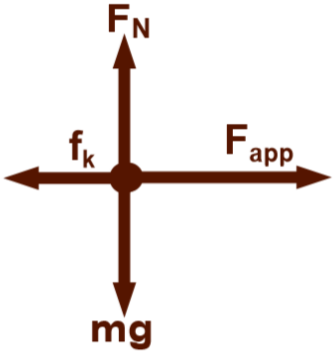
$$F_N = 30\text{N} - 12\text{N}$$

$$F_N = 18\text{N}$$

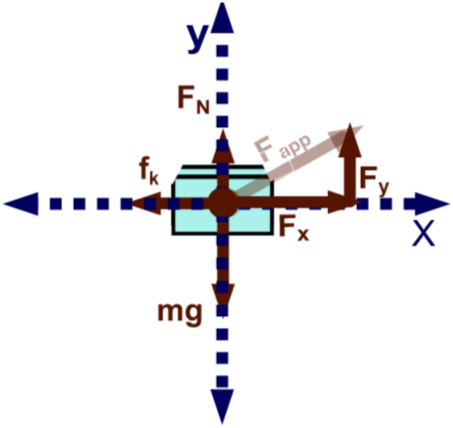
Two Dimensional Applied Force and Friction

Now it's time to compare the differences when you pull a wagon at an angle versus pulling it horizontally. Here are the givens and the free body diagrams for the two problems that were presented.

$F_{app} = F = 20.0\text{ N}$
$m = 3.0\text{ kg}$
$\mu_k = 0.20$



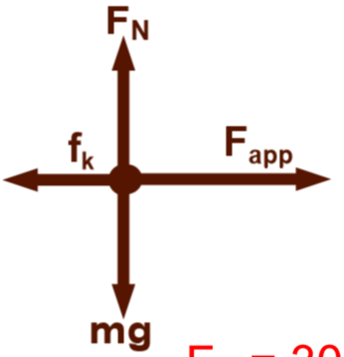
$F_{app} = F = 20.0\text{ N at }37^o$
$m = 3.0\text{ kg}$
$\mu_k = 0.20$



Two Dimensional Applied Force and Friction

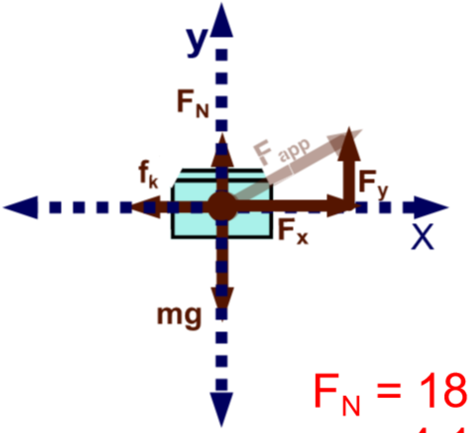
And here are the calculations for the Normal Force and the acceleration in the x direction. Can you explain the differences?

$F_{app} = F = 20.0$ N
$m = 3.0$ kg
$\mu_k = 0.20$



$F_N = 30.0$ N
$a_x = 4.7\ m/s^2$

$F_{app} = F = 20.0$ N at 37°
$m = 3.0$ kg
$\mu_k = 0.20$



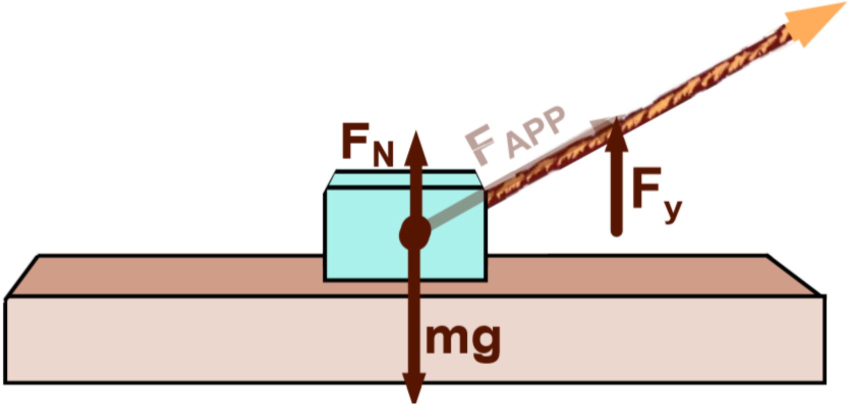
$F_N = 18.0$ N
$a_x = 4.1\ m/s^2$

Two Dimensional Applied Force and Friction

Friction was reduced when the box was pulled at an angle, because the Normal Force was reduced.

The box's weight, mg, was supported by the y-component of the Applied Force plus the Normal Force.

Thus, the Normal Force was lowered which decreased the frictional force.



Just looking at the y-axis

$$\Sigma F = ma_y$$

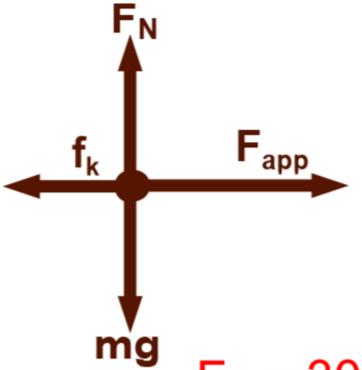
$$F_N + F_y - mg = 0$$

$$F_N = mg - F_y$$

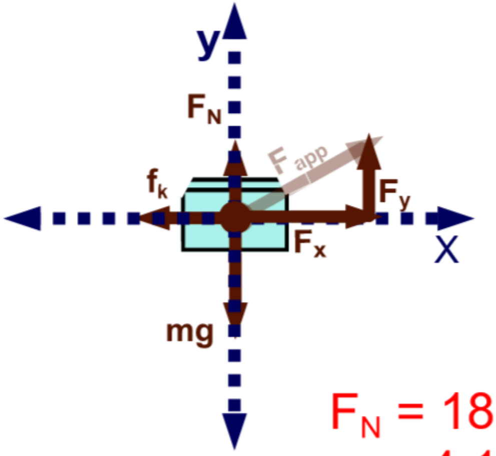
$$F_N = mg - F\sin\theta$$

Two Dimensional Applied Force and Friction

That explains the difference in the Normal force, but since the friction force is greater for the object pulled along the x axis, why is its acceleration greater than the object pulled at an angle?



$F_N = 30.0$ N
$a_x = 4.7$ m/s^2



$F_N = 18.0$ N
$a_x = 4.1$ m/s^2

Two Dimensional Applied Force and Friction

That's not as easy - it depends on the angle and the coefficient of kinetic friction - they work in tandem to determine the optimal angle for the greatest acceleration for the same applied force.

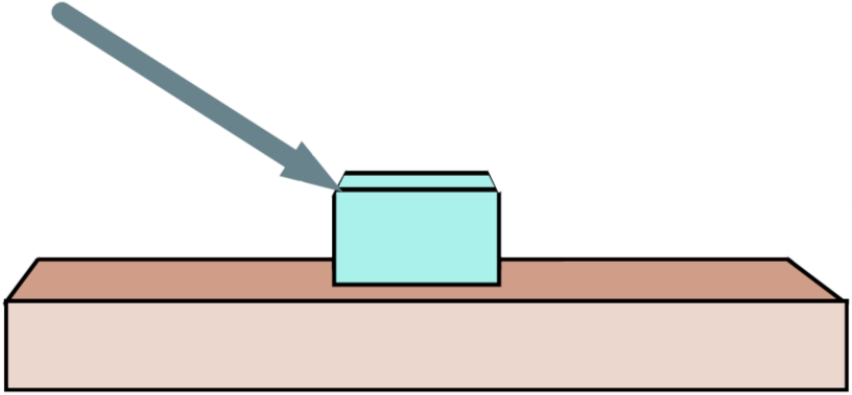
Try solving the equations algebraically for the two cases and comparing them.

You should find that if $(\cos\theta + \mu_k\sin\theta)$ is greater than 1, then pulling the object at an angle θ results in a greater acceleration than if the force is aligned with the x axis. If equal to 1, it is the same, and if less than 1, then a greater acceleration results from the force being horizontal.

Two Dimensional Applied Force and Friction

Instead of pulling the object at an angle, what would happen to the acceleration of the object if it was pushed along the floor by a downward angled force?

What would happen to the Normal force and the friction force?

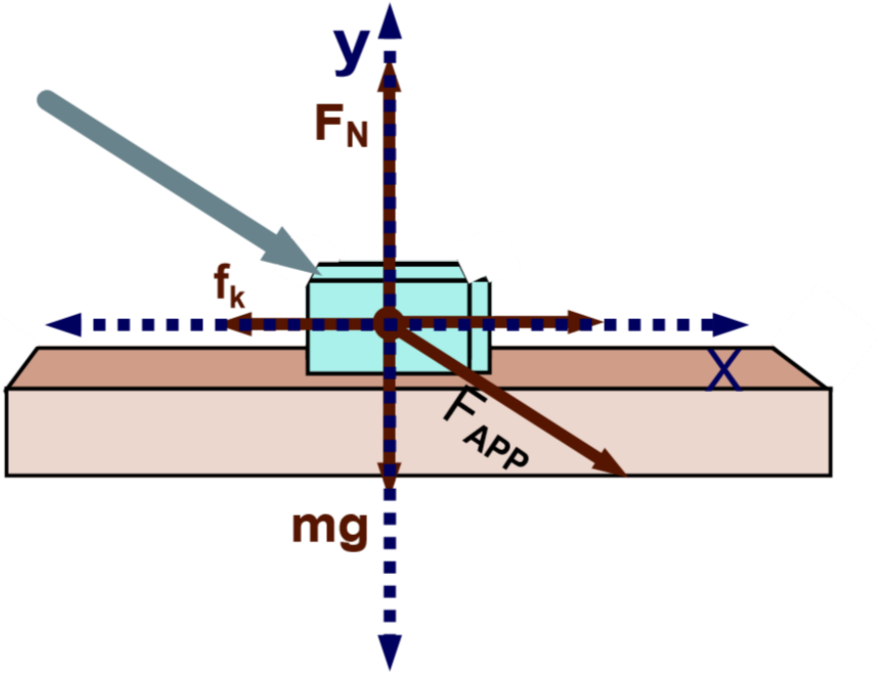


We'll just use algebra this time - no numbers.

Two Dimensional Applied Force and Friction

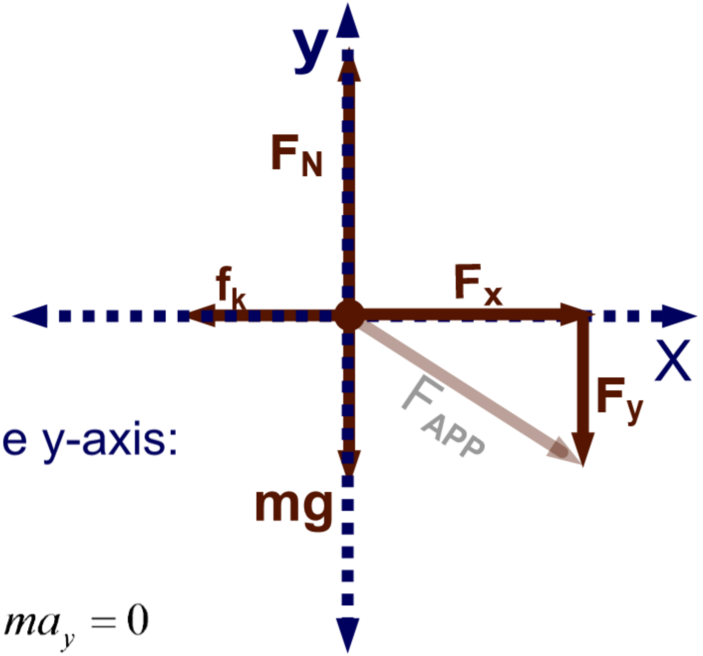
Here's the free body diagram superimposed over the box.

Next, the y components of this case will be examined.



Two Dimensional Applied Force and Friction

In this case the pushing force is also pushing the box into the surface, increasing the Normal force as well as the Friction force.



The forces on the y-axis:

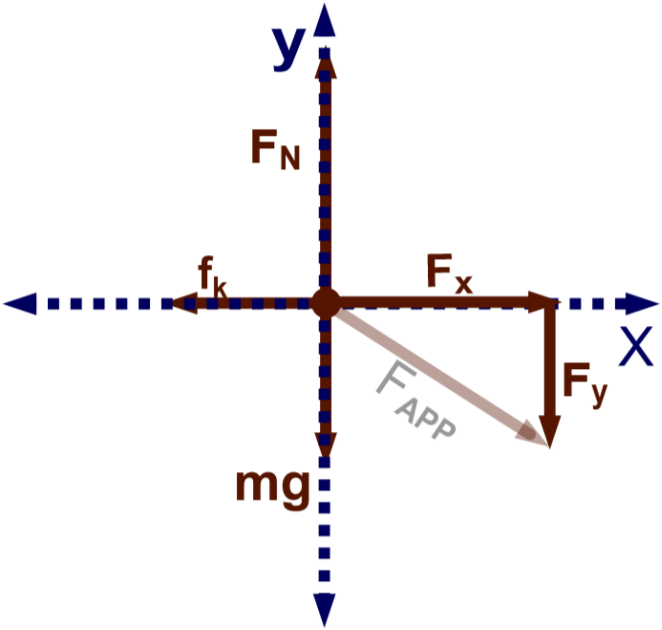
$$\Sigma F = ma_y$$

$$F_N - F_y - mg = ma_y = 0$$

$$F_N = mg + F_y$$

$$F_N = mg + F\sin\theta$$

Two Dimensional Applied Force and Friction



Now, let's look at the x axis.

The forces on the x-axis:

$$\Sigma F = ma_x$$

$$F_x - f = ma_x$$

$$F\cos\theta - \mu_k(mg + F\sin\theta) = ma_x$$

$$a_x = \frac{F\cos\theta - \mu_k(mg + F\sin\theta)}{m}$$

Two Dimensional Applied Force and Friction

The acceleration in the x direction for pushing an object along a horizontal surface is:

$$a_{xpush} = \frac{F\cos\theta - \mu_k(mg + F\sin\theta)}{m}$$

Had we done the algebra for the pulling case, the only difference is the sign of the $F\sin\theta$ term:

$$a_{xpull} = \frac{F\cos\theta - \mu_k(mg - F\sin\theta)}{m}$$

Given these equations, what can you say about the acceleration of an object when it is alternatively pushed and pulled with the same force?

Two Dimensional Applied Force and Friction

For a given force, the acceleration due to pulling is always greater than pushing it.

$$a_{xpush} = \frac{F\cos\theta - \mu_k(mg + F\sin\theta)}{m} \qquad a_{xpull} = \frac{F\cos\theta - \mu_k(mg - F\sin\theta)}{m}$$

And that is why it is easier to pull a lawnmower than to push it; remember that the next time you're in the airport and you have to wheel your luggage cart to catch a plane!

27 A block is pushed at an angle of θ with respect to the horizontal as shown below. The normal force on the block is:

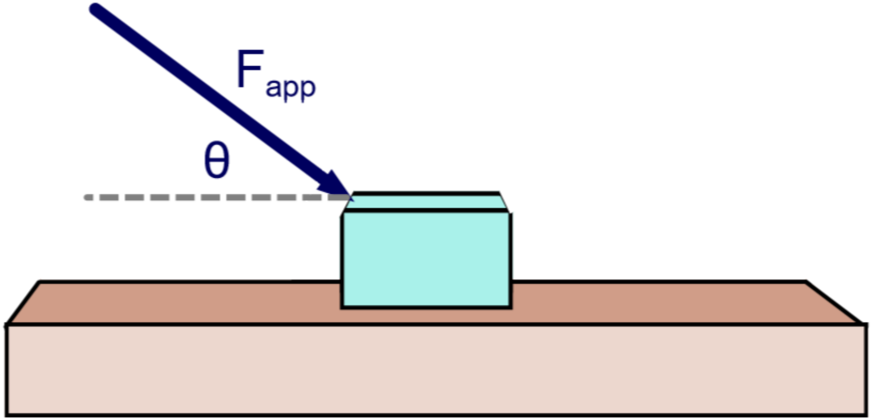
○ A mg

○ B $mg\sin\theta$

○ C $mg - F_{app}\sin\theta$

○ D $mg + F_{app}\sin\theta$

○ I need help



Answer

https://www.njctl.org/video/?v=a23d01jj9hc

28 A block is pushed at an angle of θ with respect to the horizontal as shown below. The frictional force on the block is:

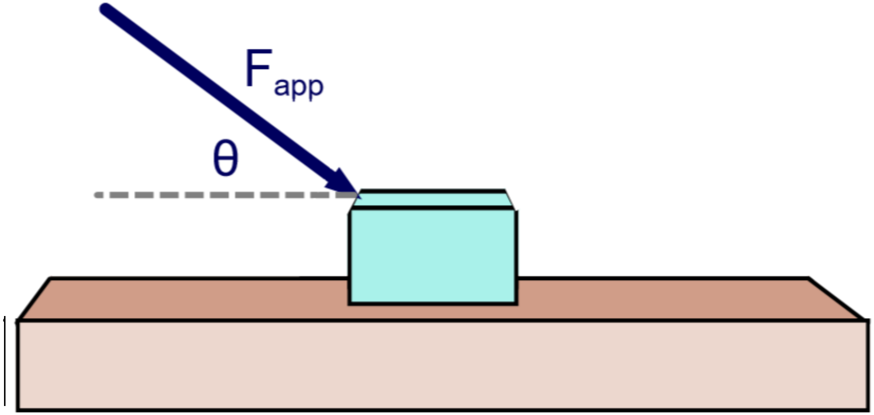
○ A $\mu_k mg$

○ B $\mu_k mg\sin\theta$

○ C $\mu_k(mg + F_{app}\sin\theta)$

○ D $\mu_k(mg - F_{app}\sin\theta)$

○ I need help



https://www.njctl.org/video/?v=nEKJ9UAuqkM

Answer

29 A block is pulled at an angle of θ with respect to the horizontal as shown below. The normal force on the block is:

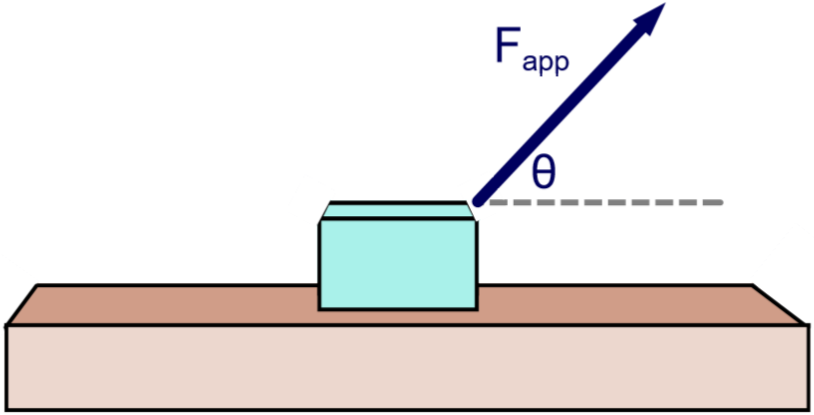
○ A mg

○ B $mg\sin\theta$

○ C $mg - F_{app}\sin\theta$

○ D $mg + F_{app}\sin\theta$

○ I need help



https://www.njctl.org/video/?v=xDW31V7WPws

Answer

The Inclined Plane

https://www.njctl.org/video/?v=JqzX73I39Lo

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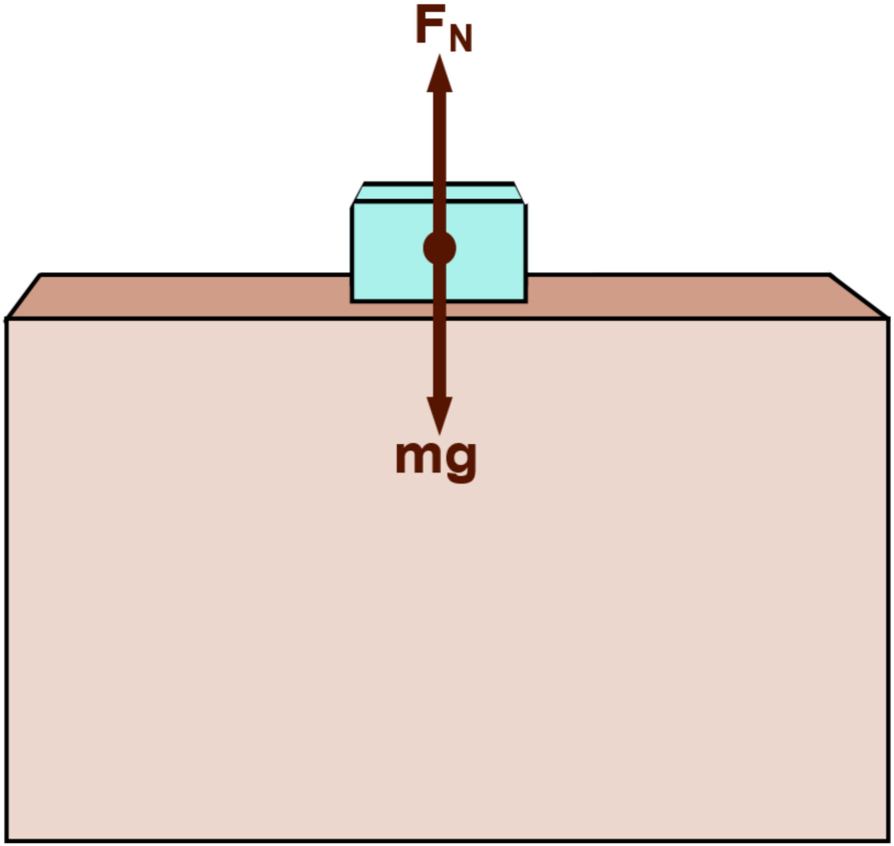
Normal Force and Weight

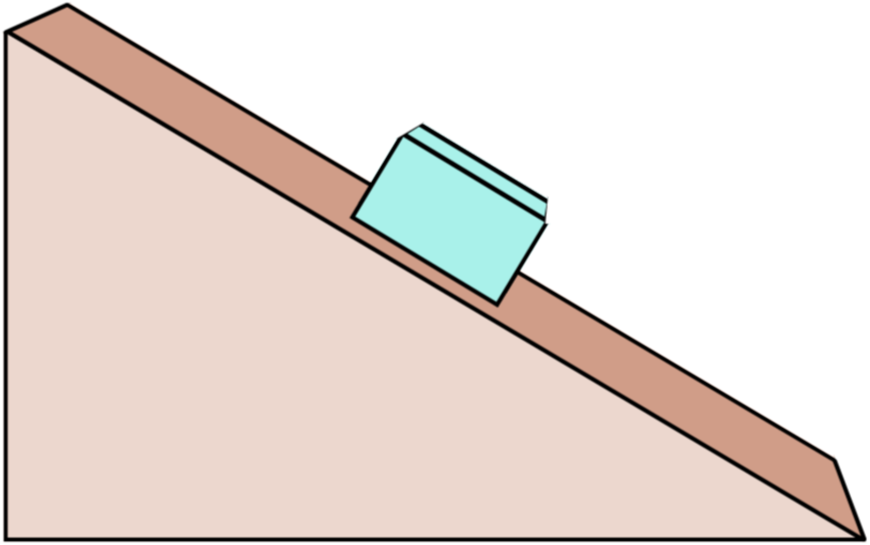
Previously we dealtwith horizontal surfaces In that case F_N and mg were always along the y axis and were equal if there was no acceleration along that axis.

Now we will look at a surface that is not horizontal.



The Inclined Plane

This non horizontal surface will be called the Inclined Plane.



On the picture, draw the free body diagram for the block.

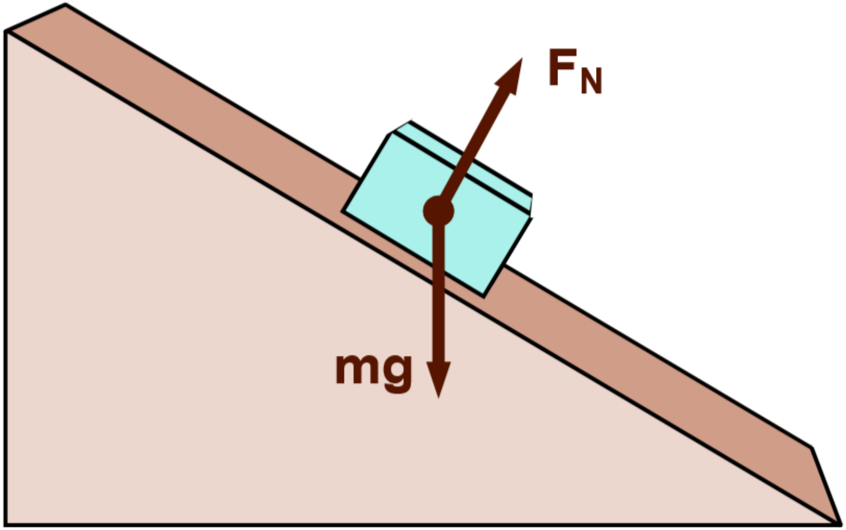
Show the weight and the normal force.

The Inclined Plane

F_N is ALWAYS perpendicular to the surface.

mg is ALWAYS directed downward.

But now, they are neither parallel nor perpendicular to one another.

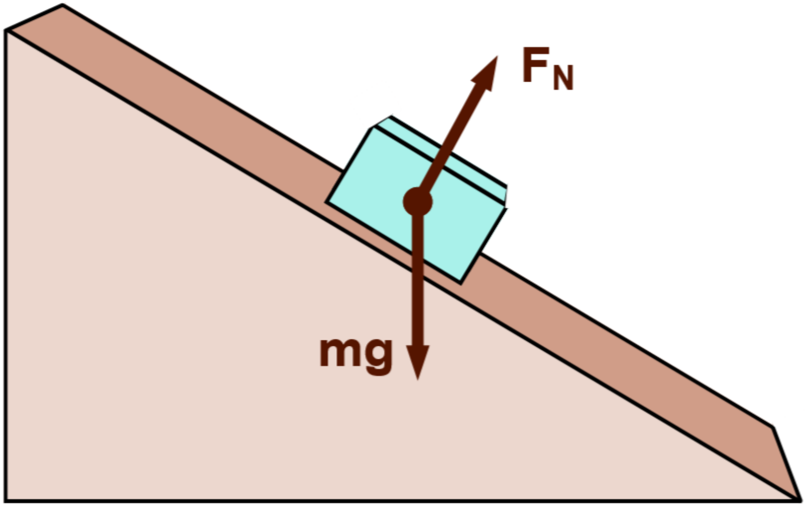


Lab

The Inclined Plane

Previously, we used horizontal and vertical (x and y) axes. That worked because problems always resulted in an acceleration that was along one of those axes.

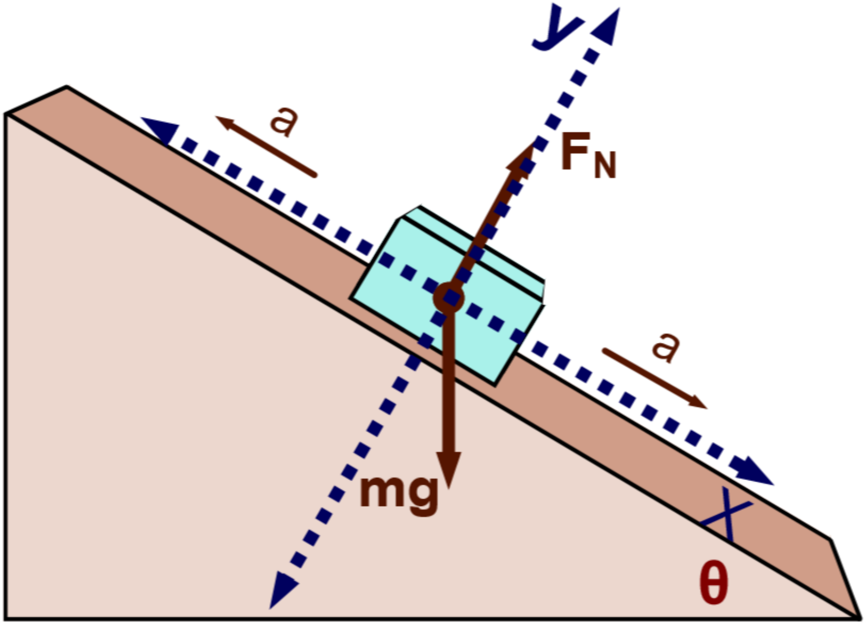
But, for the inclined plane, the acceleration is not horizontal or vertical - it will slide up or down the incline (without losing contact with the surface).



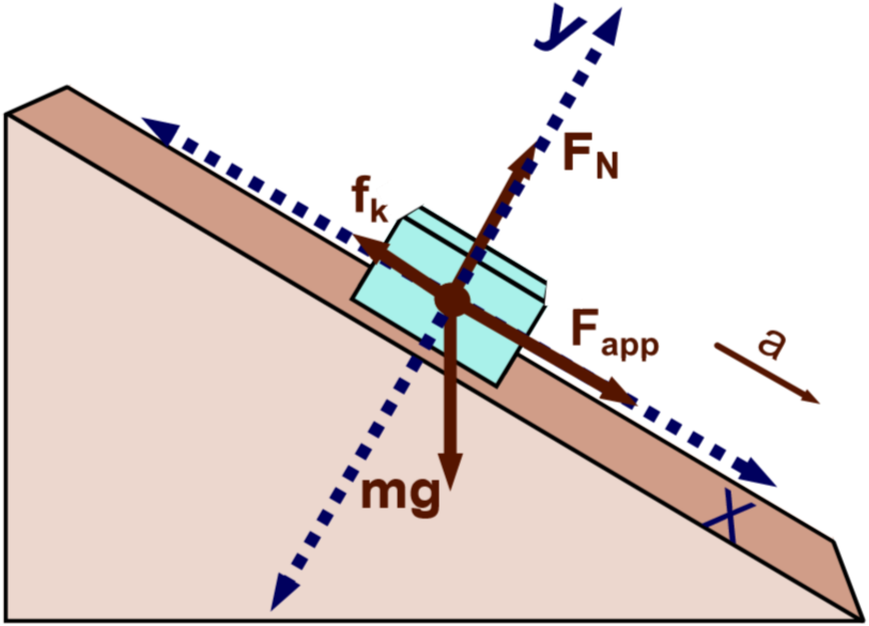
The Inclined Plane

In this case, the block can accelerate onlyalong the surface of the plane (it will slide down in the absence of an applied force - but it will go up if it is pulled).

So the x and y axes are rotated to line up with the surface of the plane.



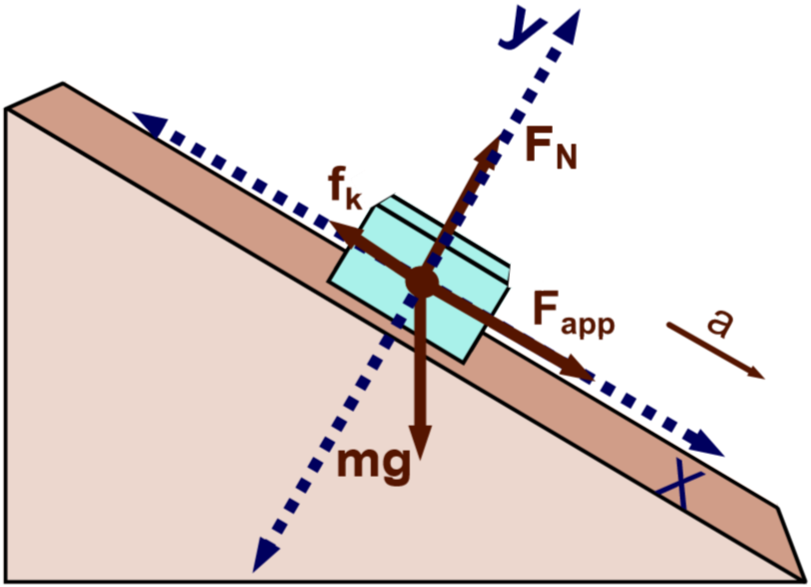
The Inclined Plane



Another reason the x and y axes were rotated is to simplify the mathematics - especially when the friction force is added.

Assume F_{app} is pulling the block down the incline and f_k is the force of kinetic friction.

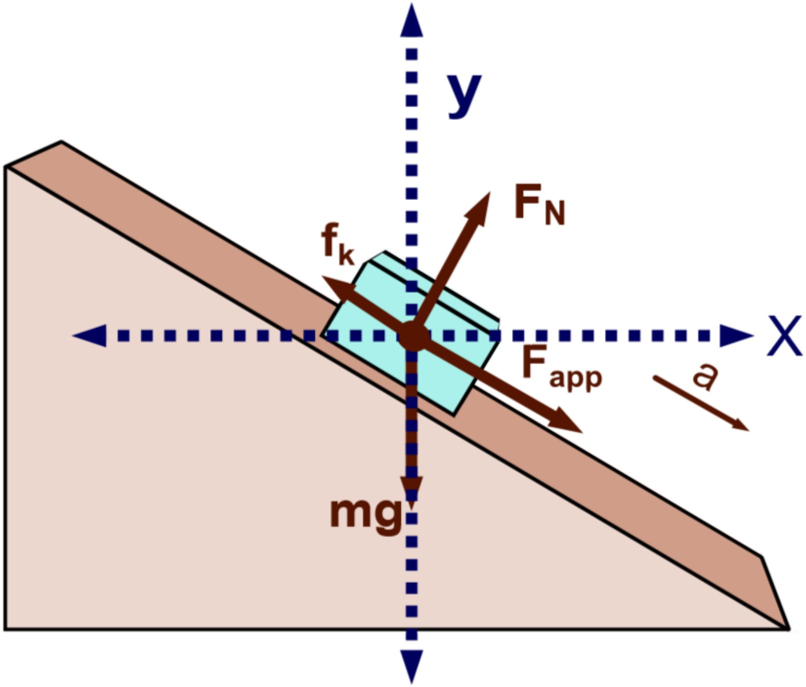
The Inclined Plane



By rotating the coordinate system, we only need to resolve one force (mg) into its x and y components.

Let's assume that you don't want to rotate the axes because it is a new concept to you. How many forces would you have to resolve into x and y components?

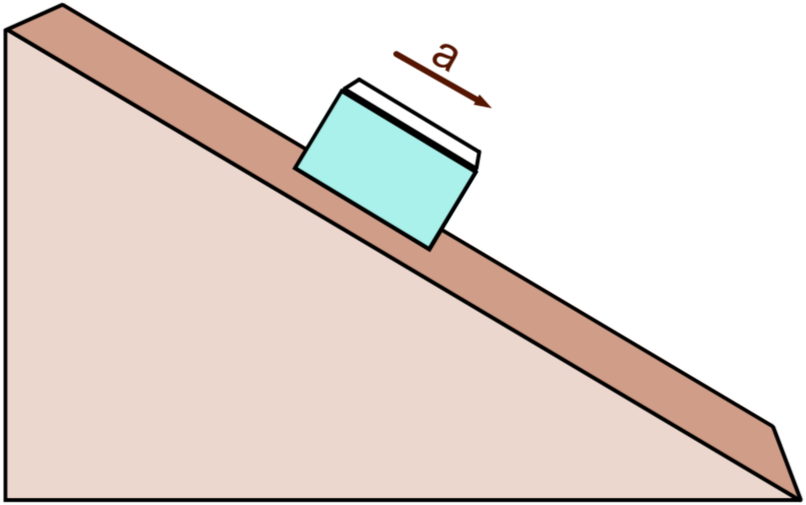
The Inclined Plane



If the x and y axes were the conventional horizontal/ vertical setup, we'd have three forces (F_N, F_{app} and f_k) to resolve into their components along those axes!

That's three times as much trigonometry. Always something to be avoided.

A block is being pulled down a rough incline as shown below. Draw all the forces acting on the block.



Answer

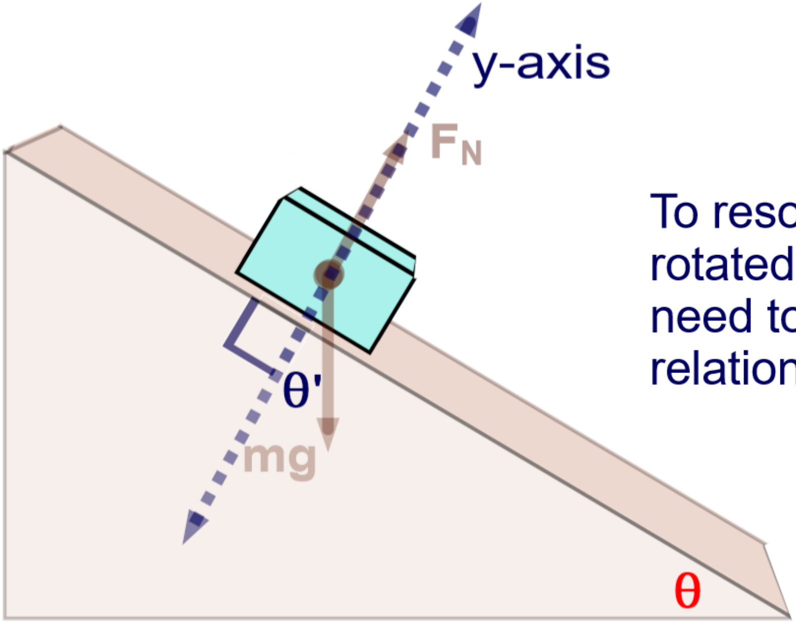
What is the purpose of rotating the normal x and y axes to align with the surface of the inclined plane?

https://www.njctl.org/video/?v=bN6QiR0kKYU

Answer

Solving the Inclined Plane

The only force not aligned with the rotated coordinate system is mg. It makes an angle of θ' with the rotated y axis. These types of problems typically give the value of θ - the angle of the incline.



To resolve mg along the rotated x and y axes, we need to find the relationship of θ' to θ.

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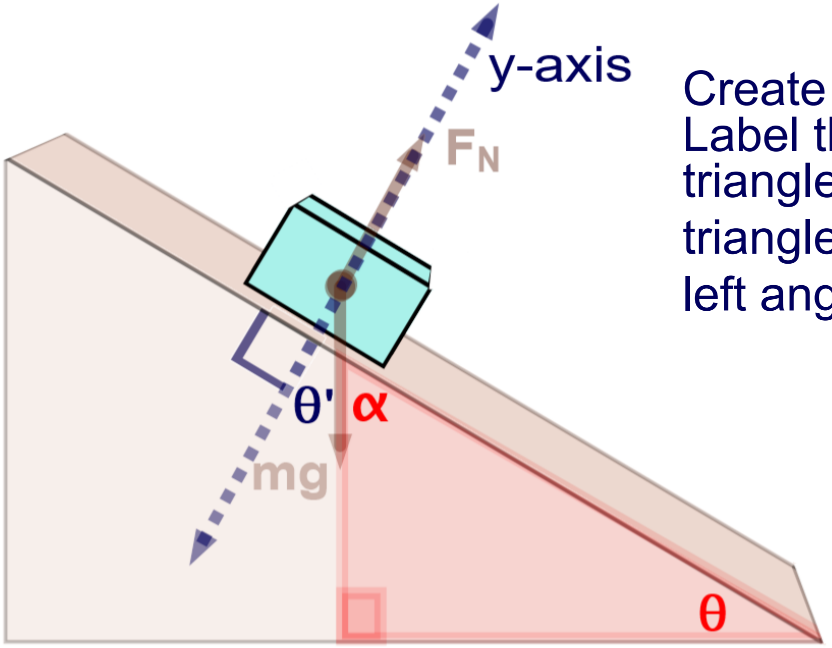
Solving the Inclined Plane

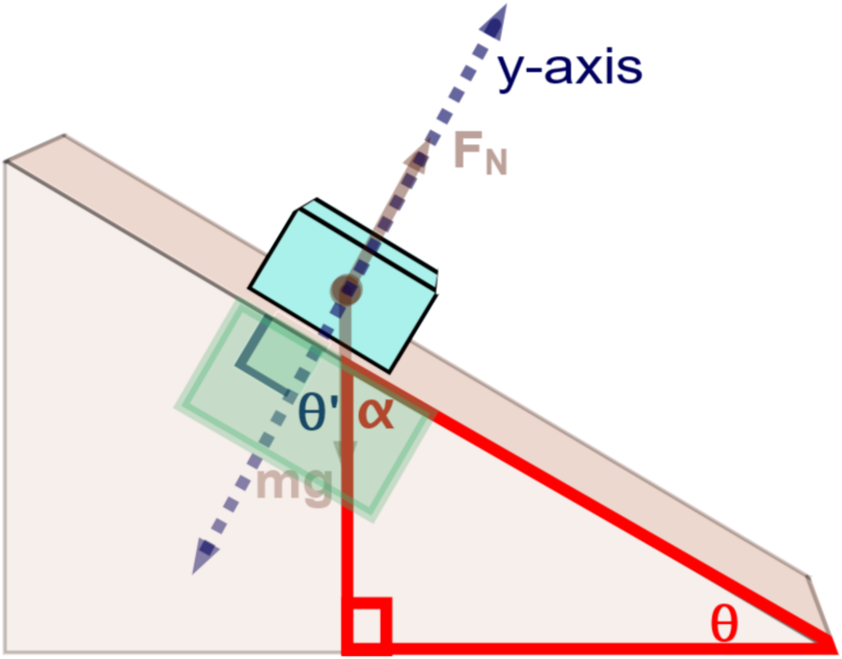


Create the red triangle as shown. Label the third angle in the red triangle as α. Since the angles in a triangle add to 180°, and the bottom left angle is 90°, that means:

$$\alpha + \theta = 90^o$$

Solving the Inclined Plane

Examine the angles at the upper left corner of the red triangle.



Since we have a right angle between the y axis and the surface, the angle α in the triangle complements the angle θ' from the y axis:

$$\theta' + \alpha = 90^o$$

But we already showed that

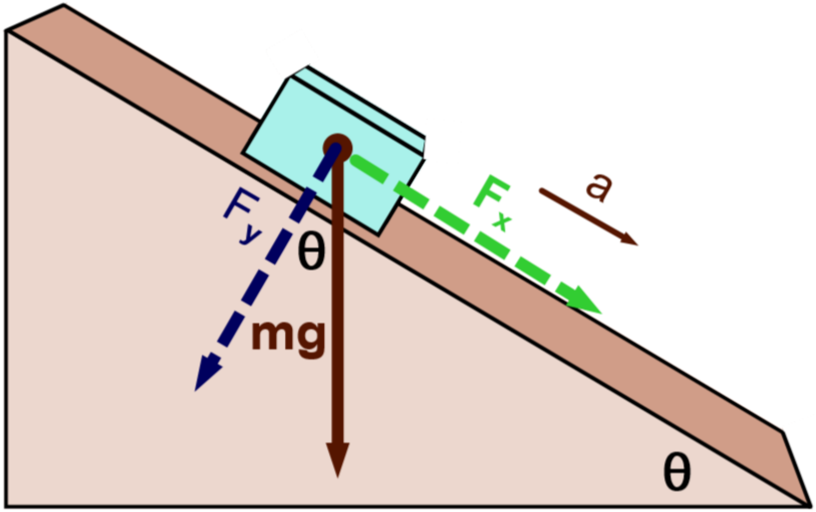
$$\alpha + \theta = 90^o$$

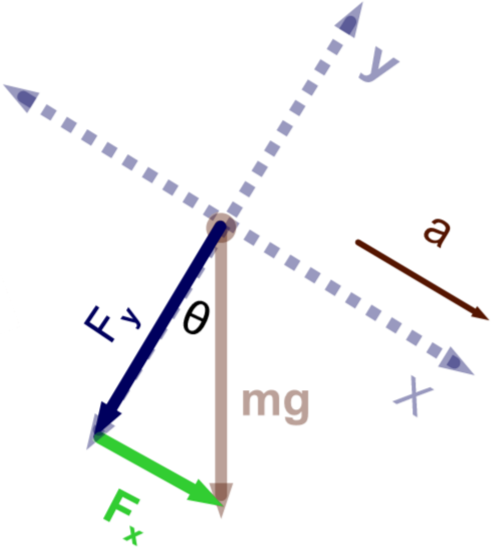
So we can conclude:

$$\theta' = \theta$$

Solving the Inclined Plane

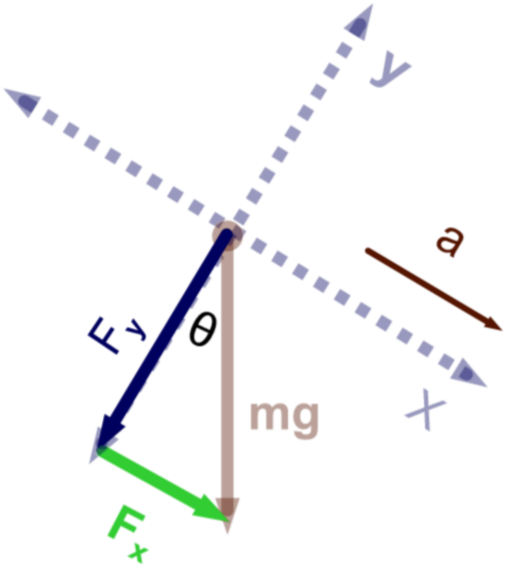
The gravitational force (mg) has components into the surface (y) and ALONG the surface (x) and will affect the force equations in both cases. To determine its impact, it needs to be resolved into its x and y components along the rotated axes.





Solving the Inclined Plane

As done earlier in this chapter, trigonometry will be used to resolve mg into its components along the rotated x and y axes.



F_x is opposite θ, so the sine function will be used:

$$\sin\theta = {}^{opp}\!/_{hyp}$$

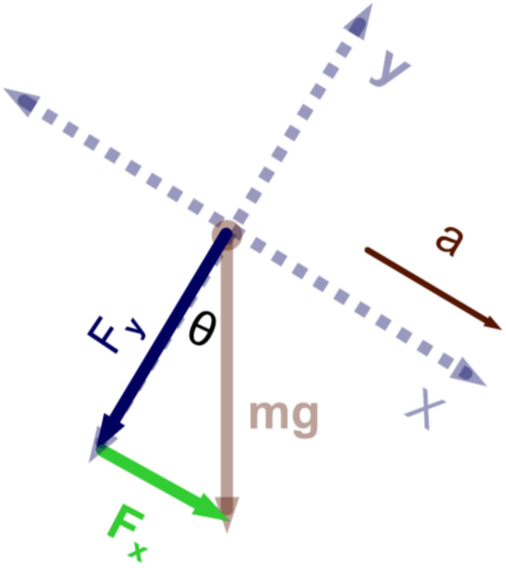
$$\sin\theta = \frac{F_x}{mg}$$

$$F_x = mg\sin\theta$$

F_x is the component of mg on the x axis and will accelerate the box down the incline.

Solving the Inclined Plane

Next, solve for the y component of the gravitational force.



F_y is adjacent to θ, so the cosine function will be used:

$$\cos\theta = {}^{adj}\!/_{hyp}$$

$$\cos\theta = \frac{F_y}{mg}$$

$$F_y = mg\cos\theta$$

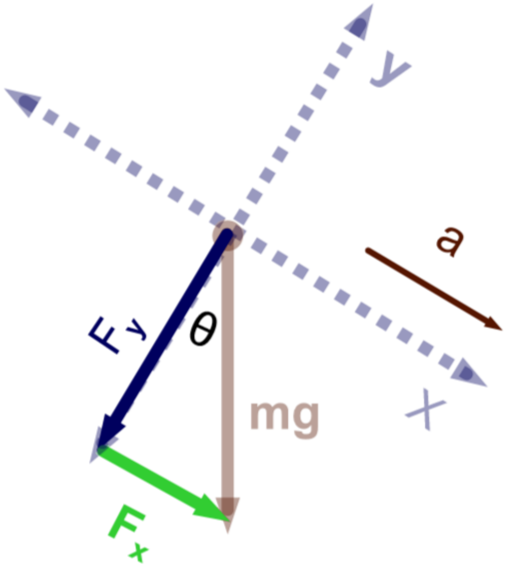
F_y is the component of mg on the y axis and will be equal to the normal force.

Solving the Inclined Plane

The gravitational force has now been resolved into x and y components along the rotated

$$F_y = mg\cos\theta$$

$$F_x = mg\sin\theta$$



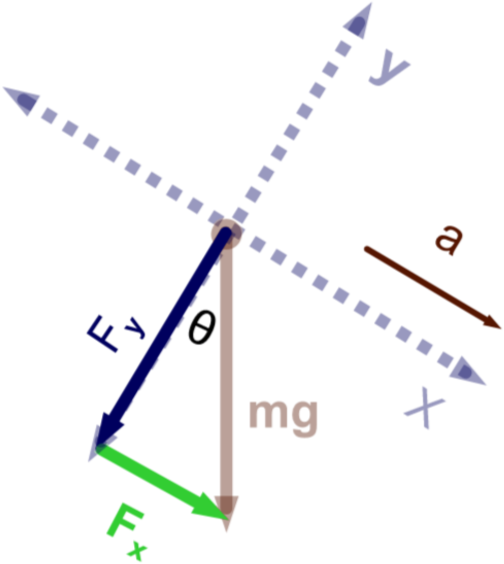
How is this different from when we would resolve forces along the non rotated coordinate system - where the x axis was horizontal and the y axis was vertical?

Solving the Inclined Plane

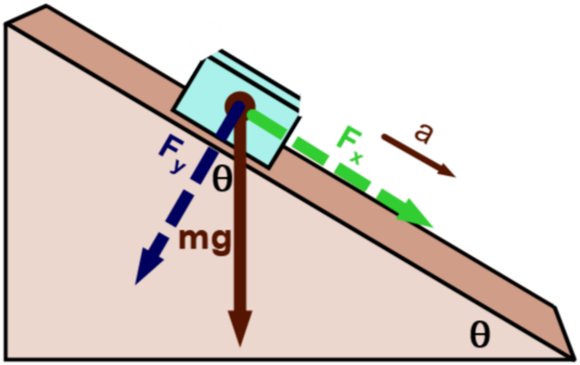
In the non rotated coordinate system, we were used to seeing force and acceleration components using the cosine function in the x direction and the sine function in the y direction. That's reversed now.

$$F_y = mg\cos\theta$$

$$F_x = mg\sin\theta$$



30 Given an inclined plane that makes an angle of 32^0 with the horizontal, and a box of 5.8 kg. What is the component of the gravitational force along the incline?

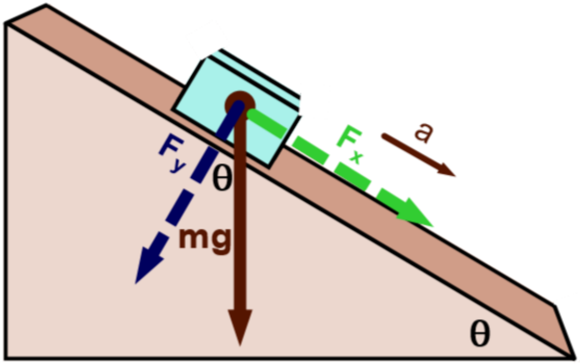


- ○ A 31 N
- ○ B 38 N
- ○ C 45 N
- ○ D 51 N
- ○ E I need help

https://www.njctl.org/video/?v=I7QJcFKapQA

Answer

31 Given an inclined plane that makes an angle of 32^0 with the horizontal, and a box of 5.8 kg. What is the component of the gravitational force into the incline?



○ 31 N

○ 38 N

○ 45 N

○ 49 N

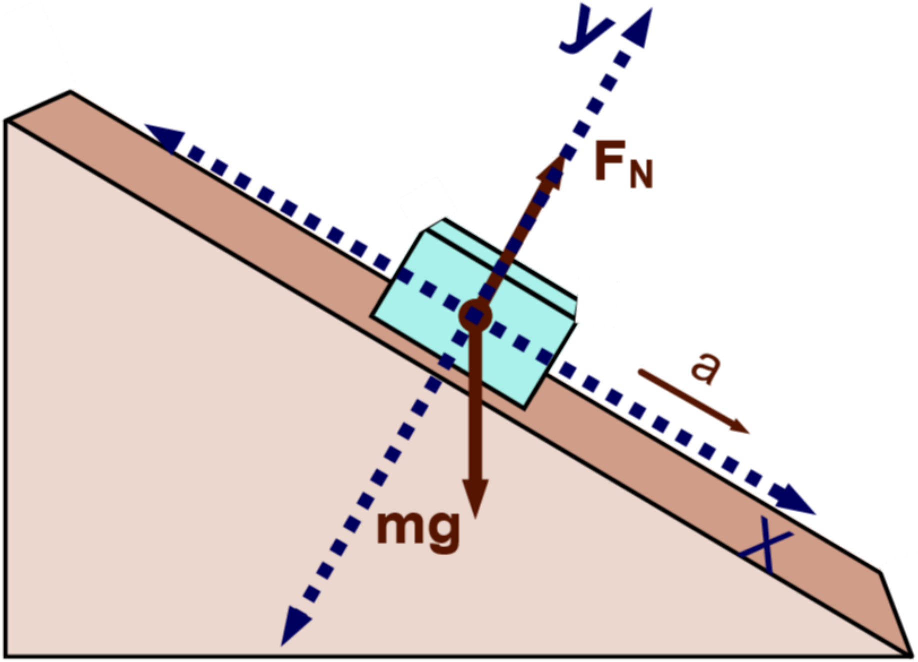
○ I need help

https://www.njctl.org/video/?v=WRzLJkHd-Fw

Answer

Putting it all together

Now that mg has been resolved into its two components along the roatated x and y axes, the Normal force will be added and the total frictionless picture will be shown.



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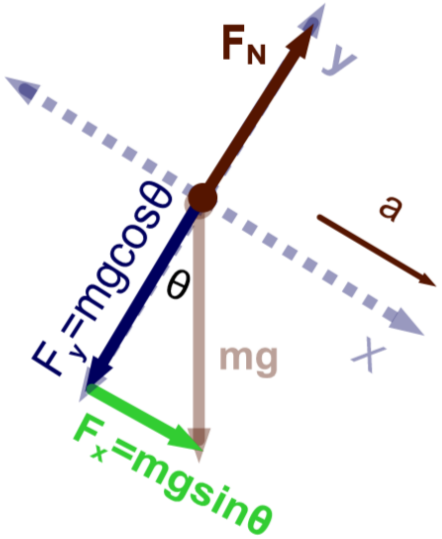
Putting it all together

Apply Newton's Second Law to the forces on each rotated axis



x - axis	y - axis
$\Sigma F_x = ma_x$	$\Sigma F_y = ma_y = 0$
$mg\sin\theta = ma_x$	$F_N - mg\cos\theta = 0$
$a_x = g\sin\theta$	$F_N = mg\cos\theta$

Note that a_x does not depend on the mass of the object! And $\Sigma F_y = 0$ since the block is sliding down the incline without bouncing.

32 What is the x component of the gravitational force?

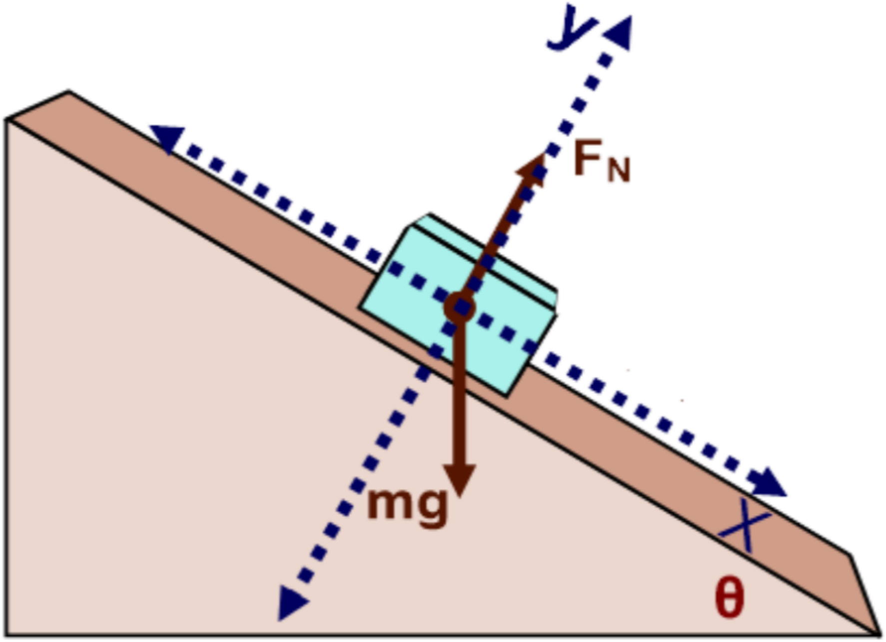
○ A mg cosθ

○ B mg sinθ

○ C mg tanθ

○ D mg

○ E I need help



Answer

https://www.njctl.org/video/?v=AWve9amv9jQ

33 What is the y component of the gravitational force?

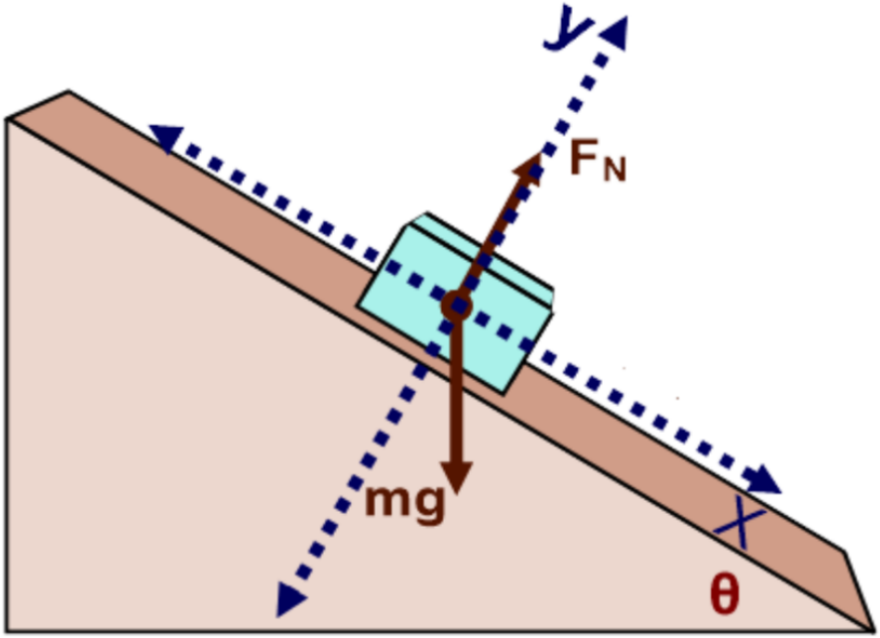
○ A mg cosθ

○ B mg sinθ

○ C mg tanθ

○ D mg

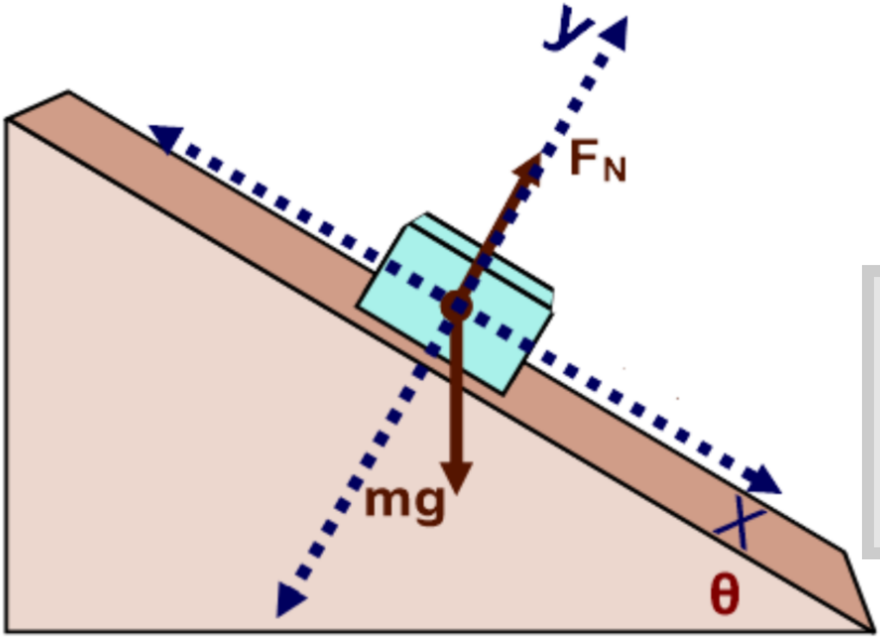
○ E I need help



https://www.njctl.org/video/?v=BFB-zjkcnBY

Answer

34 Determine the values of F_N and the x and y components of the gravitational force, given m = 20.0 kg and $\theta = 40.0^0$.



○ A 150 N, 126 N, 150 N

○ B 150 N, 160 N, 120 N

○ C 200 N, 120 N, 160 N

○ D 200 N, 110 N, 150 N

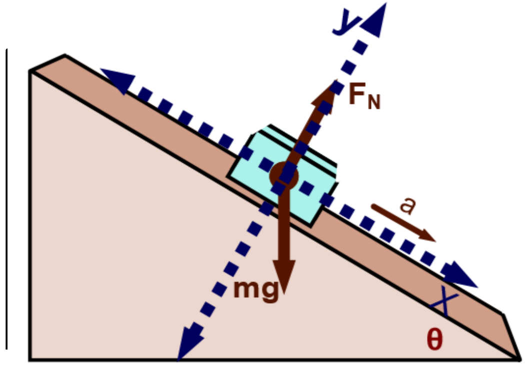
○ E I need help

https://www.njctl.org/video/?v=A0vqX9A-6n4

Answer

35 A 5 kg block slides down a frictionless incline at an angle of 30^0. Draw a free body diagram and find its acceleration.

Use g = 10 m/s^2.



○ A 2 m/s^2

○ B 3 m/s^2

○ C 5 m/s^2

○ D 10 m/s^2

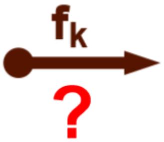
○ E I need help

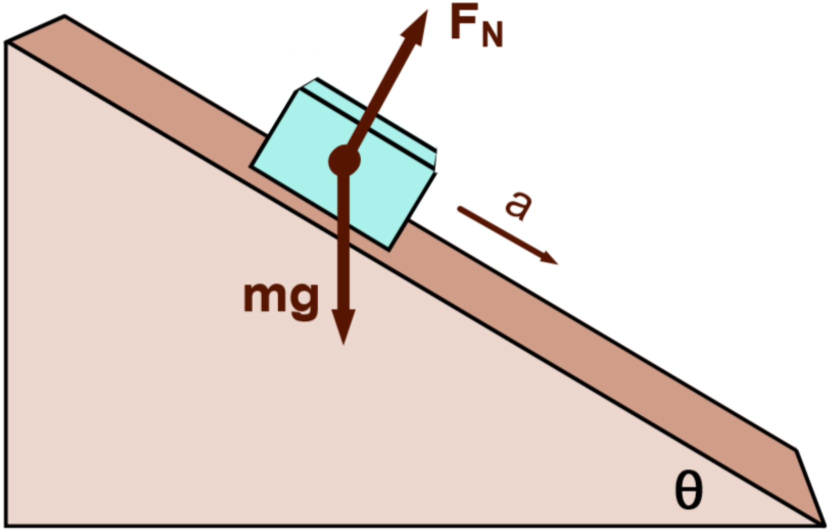
https://www.njctl.org/video/?v=iv3tI9lwVMg

Answer

Solving the Inclined Plane with Friction

It's now time to make the problem more realistic by noting that not too many surfaces are frictionless. Let's assume there is friction between the plane and the box and it is moving down the incline. What is the direction of the kinetic friction force?





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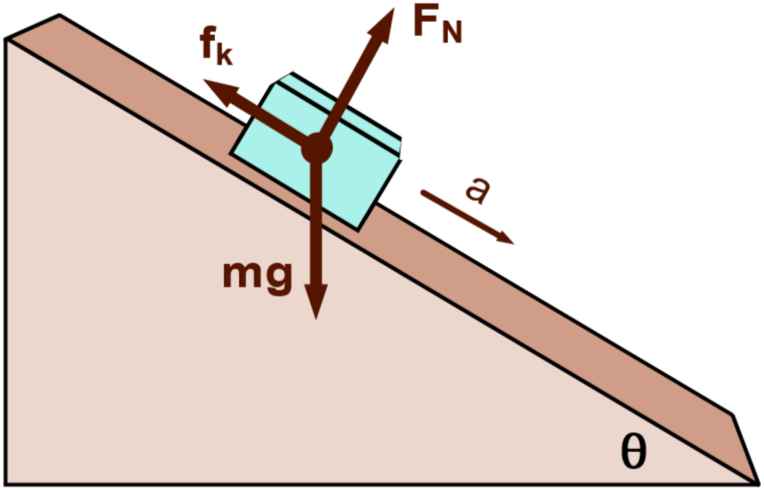
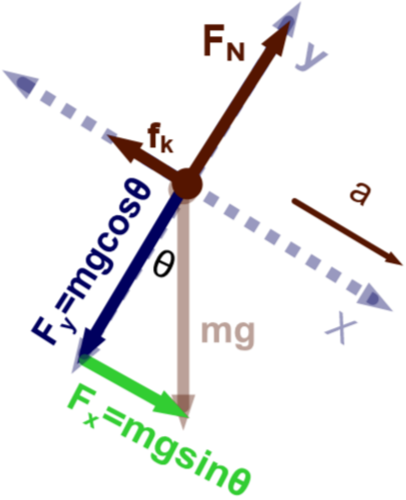
Solving the Inclined Plane with Friction

Since kinetic friction acts to oppose motion, it is directed up the incline in the negative x direction ($f_k = \mu_k F_N$)





Solving the Inclined Plane with Friction

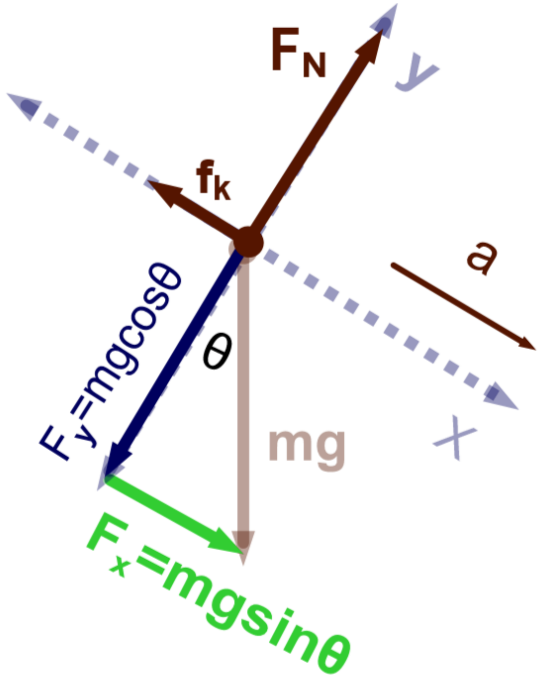
x - axis

$\Sigma F_x = ma_x$
$mg\sin\theta - f_k = ma_x$
$mg\sin\theta - \mu_k F_N = ma_x$
$mg\sin\theta - \mu_k mg\cos\theta = ma_x$
$g\sin\theta - \mu_k g\cos\theta = a_x$
$a_x = g\sin\theta - \mu_k g\cos\theta$
$a_x = g(\sin\theta - \mu_k \cos\theta)$

Remember to solve the y-axis forces first to find F_N which is required to determine the friction force on the x-axis.

y - axis

$\Sigma F_y = ma_y = 0$
$F_N - mg\cos\theta = 0$
$F_N = mg\cos\theta$



Solving the Inclined Plane with Friction

The general solution for objects sliding down an incline is:

$$a_x = g(\sin\theta - \mu_k \cos\theta)$$

This is a bit different from the result that we obtained for a_x when there was no friction ($a_x = g\sin\theta$).

A common practice in physics is to apply a limiting case to an answer and see if it gives you something that agrees with previous work. Typical examples are letting a variable equal zero or approach infinity.

Which limiting variable would you use here to validate the acceleration for an inclined plane with friction?

Solving the Inclined Plane with Friction

For the inclined plane without friction, you assume that:

$$\mu_k = 0$$

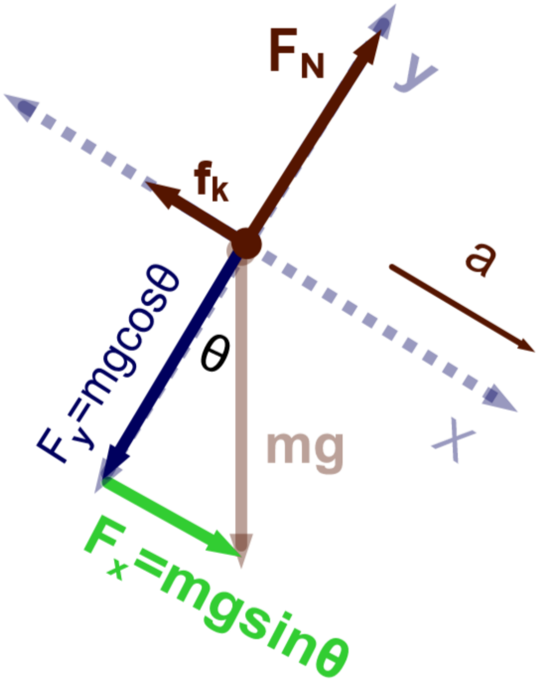
Plug this into: $a_x = g(\sin\theta - \mu_k\cos\theta)$

and we get the previous result for a frictionless inclined plane:

$$a_x = g\sin\theta$$

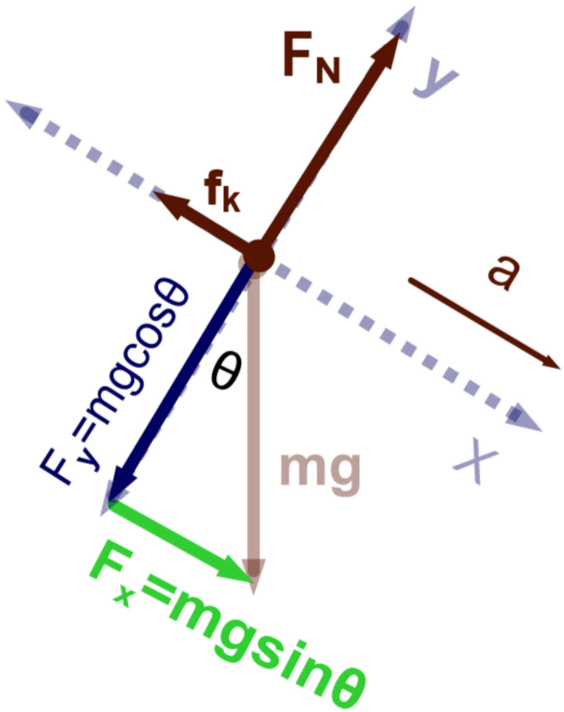
So, the equation checks out with previous knowledge. Of course, that doesn't prove it's right, but it is a good indicator that we're at least on the right track (in this case, we are right!)

Solving the Inclined Plane with Friction



What if we were given an inclined plane at an angle with the horizontal and wanted to find what value of the coefficient of kinetic friction would allow the block to slide down with a constant velocity?

Solving the Inclined Plane with Friction



We would use the previously calculated a_x equation and set it equal to zero.

$$a_x = g(\sin\theta - \mu_k\cos\theta)$$

$$0 = g(\sin\theta - \mu_k\cos\theta)$$

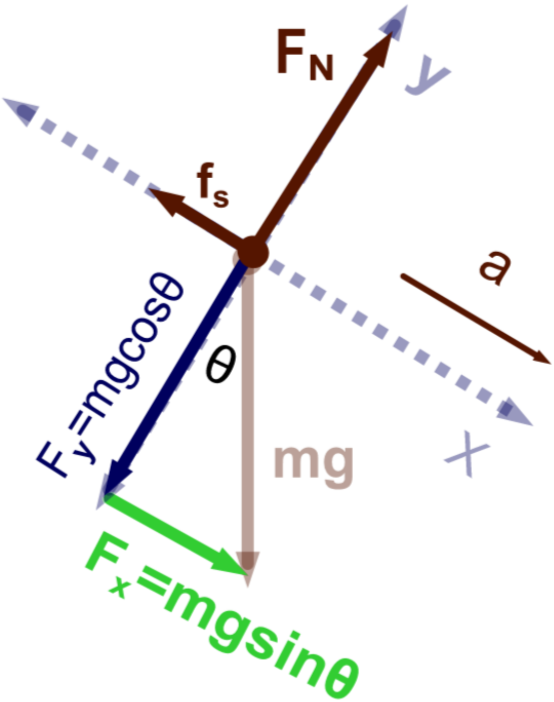
$$0 = \sin\theta - \mu_k\cos\theta$$

$$\mu_k \cos\theta = \sin\theta$$

$$\mu_k = \sin\theta / \cos\theta$$

$$\mu_k = \tan\theta$$

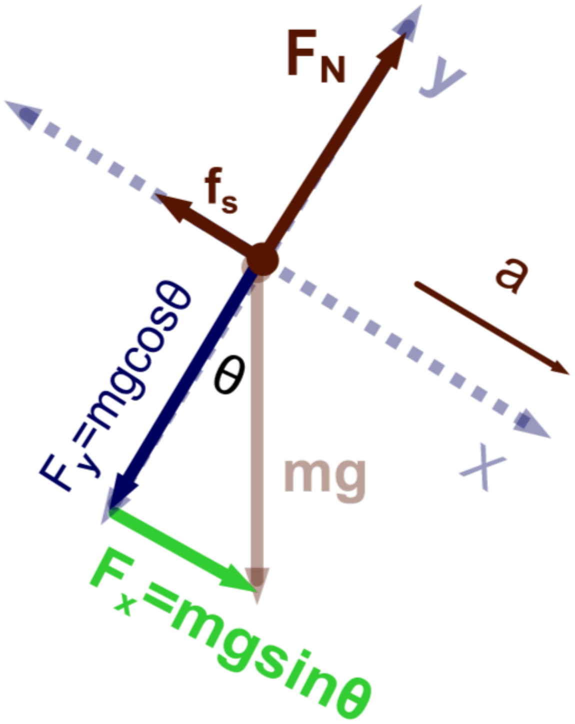
Solving the Inclined Plane with Friction



Let's change the problem a little. Assume the object is stationary on an inclined plane (with friction) and we want to find the angle at which the object will start accelerating down the plane.

We now will use static friction.

Solving the Inclined Plane with Friction



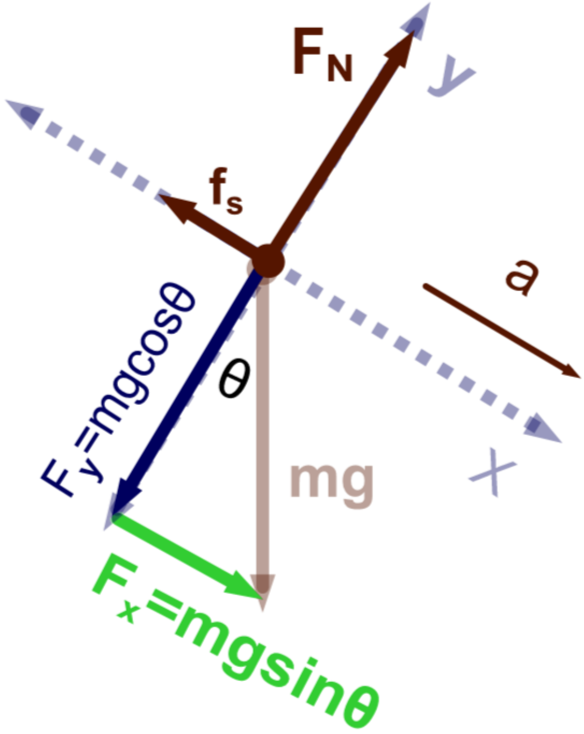
It was just shown that for an object sliding with constant velocity down an inclined plane that:

$$\mu_k = \tan\theta$$

The same free body diagram is used for the case of static friction. The only difference is that where $f_k = \mu_k N$ for kinetic friction, $f_s \leq \mu_s N$ for static friction.

How will this modify our answer?

Solving the Inclined Plane with Friction

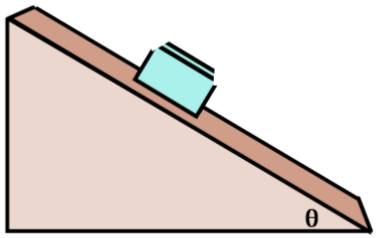


Since we're trying to find the minimum angle at which the box will start moving (accelerating), it would be when μ_s is a maximum (right before the object moves):

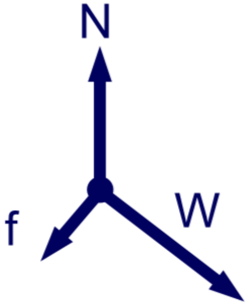
$$\mu_s = \tan\theta$$

So, the answer is - it is the same algebraic formula but μ_k is replaced by μ_s.

36 A block slides down an incline with acceleration a. Which choice represents the correct free body diagram?



A



B

○ C

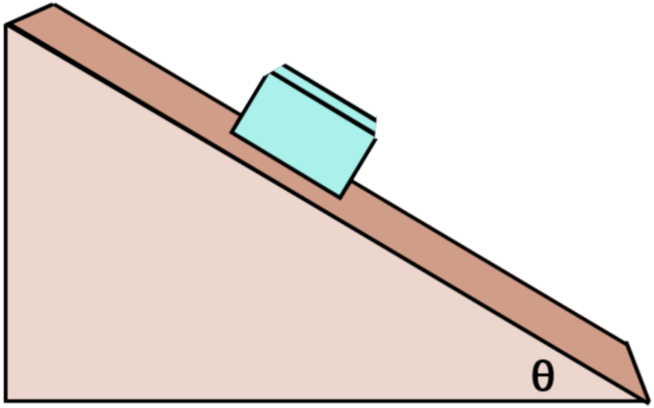
D

E I need help

Answer

https://www.njctl.org/video/?v=D350qGuIqpQ

37 A 5.0 kg block slides down an incline at an angle of 30.0⁰ with a constant speed. Draw a free body diagram and find the coefficient of kinetic friction between the block and the incline.



○ A 0.31

○ B 0.46

○ C 0.58

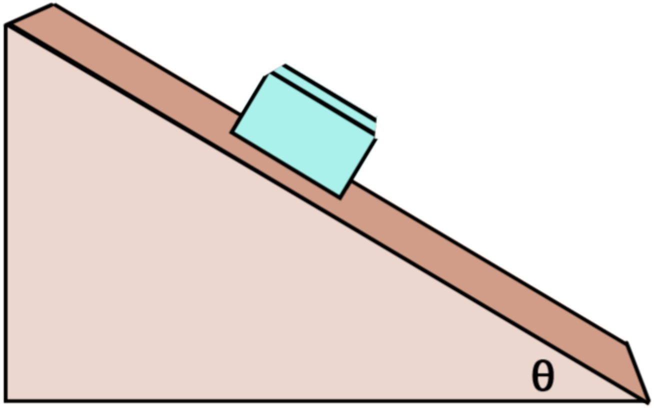
○ D 0.70

○ E I need help

https://www.njctl.org/video/?v=Dwj3ek6l5oo

Answer

38 A 7.5 kg block on an incline only starts moving when the incline angle is increased to 35^0. Draw a free body diagram and find the coefficient of static friction between the block and the incline.



○ A 0.41

○ B 0.56

○ C 0.62

○ D 0.70

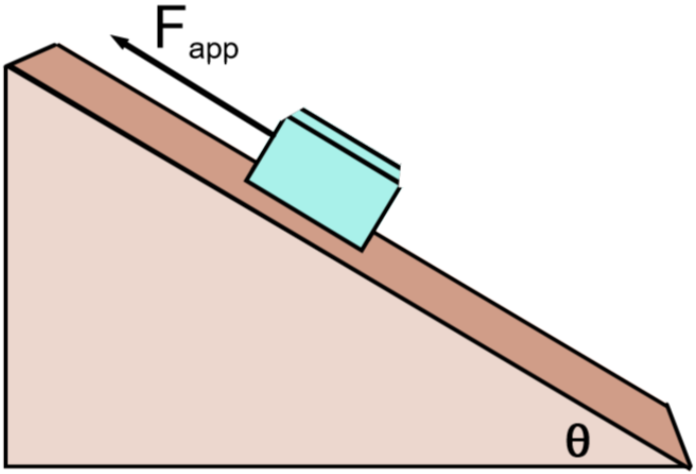
○ E I need help

Answer

https://www.njctl.org/video/?v=YR-CSKJZ9wk

39 A 5 kg block is pulled up an incline at an angle of 30^0 with a force of 40 N. The coefficient of kinetic friction between the block and the incline is 0.3. Draw a free body diagram. Find the block's acceleration.

Use g = 10 m/s^2.



A 0.4 m/s^2

B 0.6 m/s^2

C 0.7 m/s^2

D 0.8 m/s^2

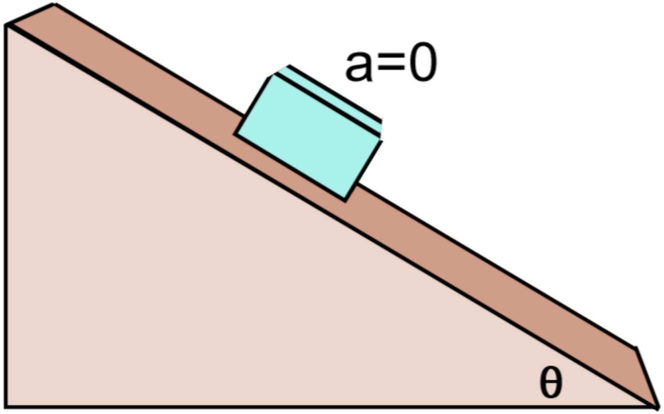
○ E I need help

https://www.njctl.org/video/?v=CrTENCc0xis

Answer

40 A 5.0 kg block remains stationary on an incline. The coefficient of static friction between the block and the incline is 0.4. Draw a free body diagram. Determine the angle at which the block will start to move.

Use $g = 10\ m/s^2$.



https://www.njctl.org/video/?v=CrpjOBAPIpY

- ○ 22^o
- ○ 35^o
- ○ 39^o
- ○ 48^o
- ○ I need help

Answer

Static Equilibrium
Tension Force

https://www.njctl.org/video/?v=WfZyIdycM5k

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Static Equilibrium

There is a whole field of problems in engineering and physics called "Statics" that has to do with cases where no acceleration occurs and objects remain at rest.

Anytime we construct bridges, buildings or houses, we want them to remain stationary, which is only possible if there is no acceleration or no net force.

There are two types of motion that we need to consider (and in both cases, motion is to be prevented!).

What are they?

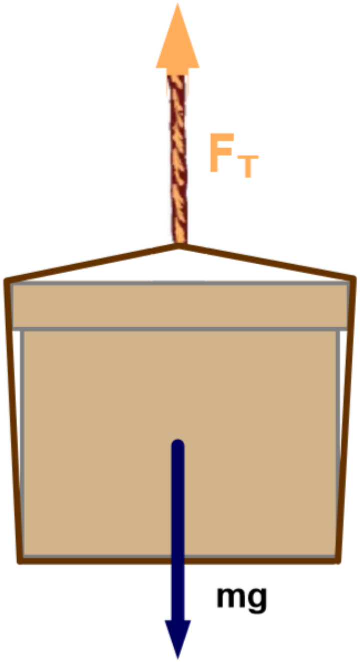
Static Equilibrium

The two types of static equilibrium relate to linear (translational) and rotational acceleration.

Linear acceleration would be when the object, or components of the object, is moving in a straight line and rotational acceleration is when it pivots about a point and rotates. Neither works well in a building or a bridge.

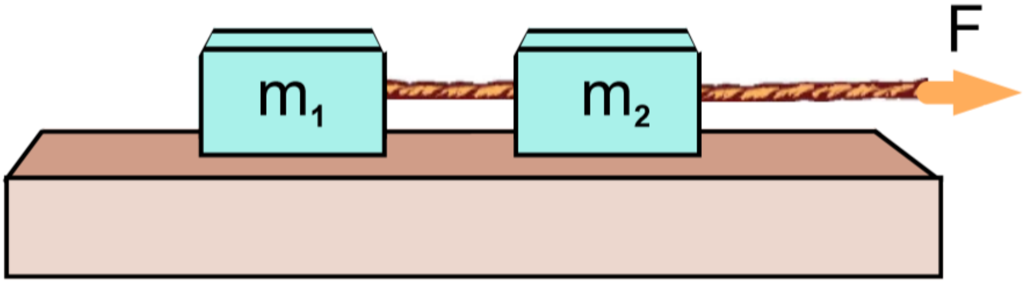
In order to prevent acceleration (movement), for the first case, the net force is equal to zero, and in the second case, the net torque is zero. Only the linear acceleration will be covered in this section. Rotational equilibrium is covered in the Rotational Motion chapter.

Tension Force



Previous problems involved a rope supporting an object by exerting a vertical force straight upwards, along the same axis as the force mg that was pulling it down. That led to the simplest case that if the box is moving up or down with a constant velocity, then $a_y = 0$, and the tension force, $F_T = mg$.

Tension Force

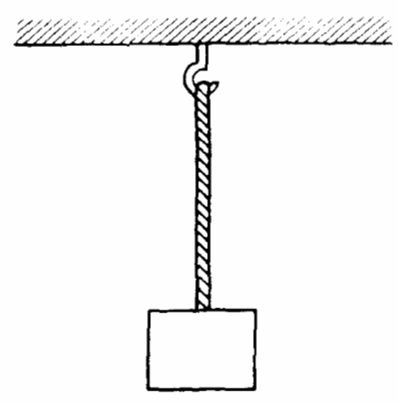


There was also the case where an applied force acted on one object, but not the other. A Tension force acted between the two masses. Again, these forces acted in the same dimension.

Let's try a few problems on this again - it was covered earlier in this chapter, but it's a good time to review.

41 A box of mass 60.0 kg is suspended from a massless rope in an elevator that is moving up, but is slowing down with an acceleration of 2.20 m/s^2. What is the tension in the rope?

Use g = 10.0 m/s^2.



○ A 311 N

○ B 389 N

○ C 468 N

○ D 515 N

○ E I need help

https://www.njctl.org/video/?v=B0NDb547d0U

Answer

42 Multi-Correct: What horizontal forces are acting on the two blocks below?

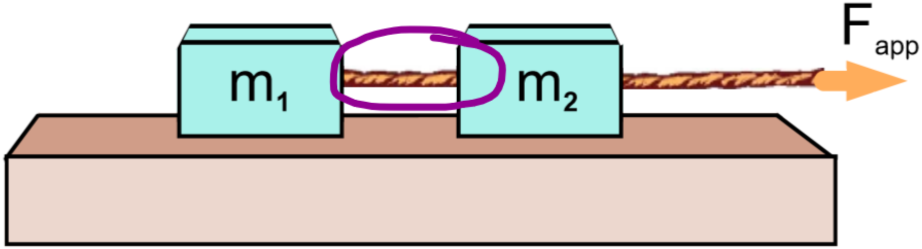
☐ Tension force on Block m_1

☐ Tension force on Block m_2

☐ Applied force on Block m_1

☐ Applied force on Block m_2

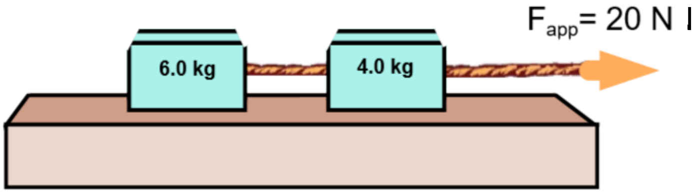
☐ E I need help



https://www.njctl.org/video/?v=IdSpwSfXRmg

Answer

43 A system of two blocks of masses 6.0 kg and 4.0 kg is accelerated by an applied force of 20.0 N on a frictionless horizontal surface. Draw a free body diagram for each block and find the Tension in the rope connecting the blocks.



○ 6 N

○ 10 N

○ 12 N

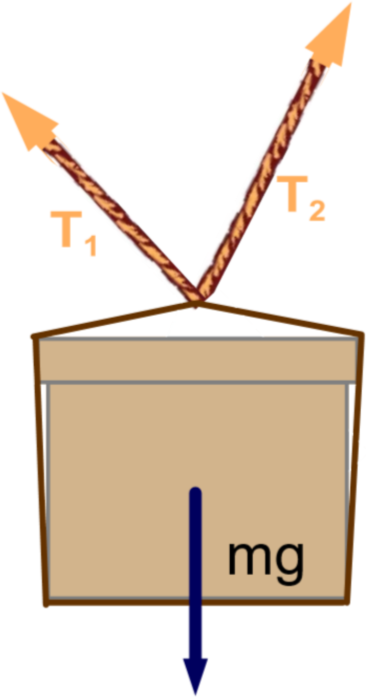
○ 18 N

○ I need help

Answer

https://www.njctl.org/video/?v=05x3TYwPeQQ

Tension Force



A more interesting problem is for two (or more) ropes to s upport a stationary object (a = 0) by exerting forces at angles.

Since we're going to focus on the Tension force for a while, and using additional subscripts (1 and 2), we'll save notation and replace F_T with T. As a physics person, you're allowed to do this!

In this case, since the it is at rest, the ΣF_x and ΣF_y on the object are zero.

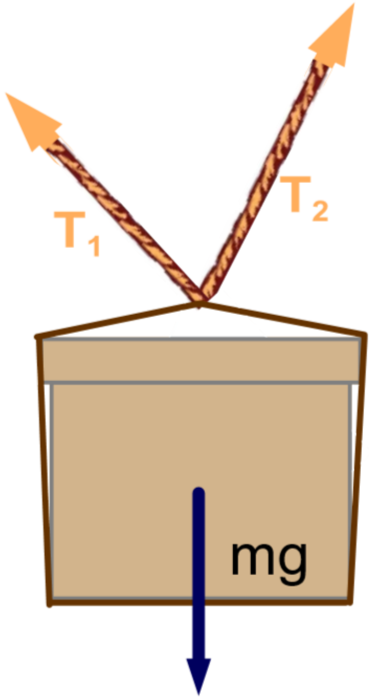
Where would you see this outside the physics classroom?

https://www.njctl.org/video/?v=DIpcR9y18HE

Tension Force

In the cables that hold traffic lights over the street.

Tension Force



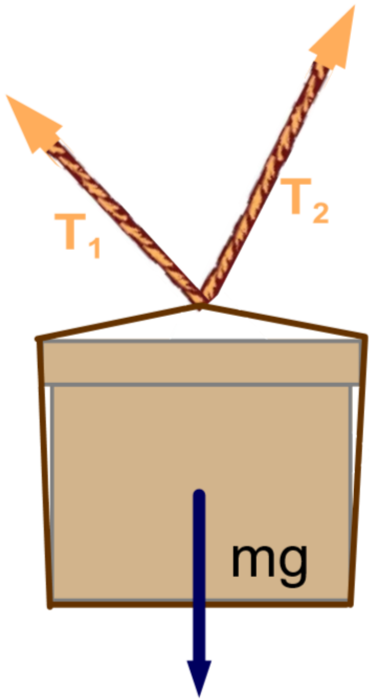
Since the only other force on the object is gravity, the vertical components of the force exerted by each rope must add up to mg. And if the object isn't moving, then $a_y = 0$.

$$\Sigma F_y = T_{1y} + T_{2y} - mg = ma_y = 0$$

$$T_{1y} + T_{2y} - mg = 0$$

$$T_{1y} + T_{2y} = mg$$

Tension Force



The only forces in the x direction are those that are provided by the x components of T_1 and T_2. Again, the object isn't moving, so $a_x = 0$.

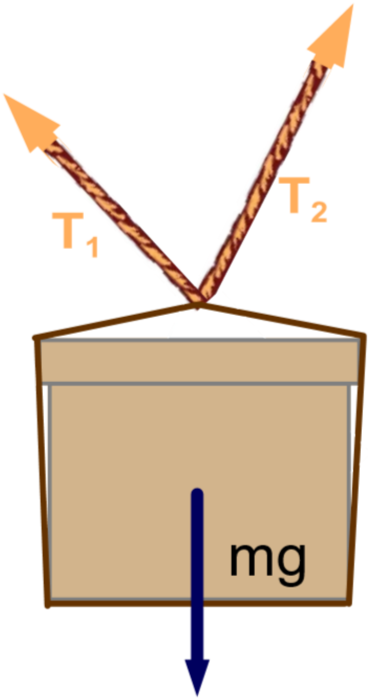
$$\Sigma F_x = -T_{1x} + T_{2x} = ma_x = 0$$

$$-T_{1x} + T_{2x} = 0$$

$$T_{1x} = T_{2x}$$

Note the negative sign on T_{1x} since it is pointing to the left (negative x axis).

Tension Force



Going forward, we will be dealing with the magnitudes of the various Tension components. We take into account the directions in the below equations, where T_{1x} was assigned a negative value as it was pointing to the left, and T_{2x}, T_{1y} and T_{2y} were all assigned positive values.

That's the great value of making a sketch and a free body diagram - you don't have to worry about angles greater than 90^0 and the different signs of the cosine and sine functions in the various graph quadrants.

$$T_{1y} + T_{2y} = mg$$

$$T_{1x} = T_{2x}$$

44 In the case of two ropes holding up a stationary bucket, what is the relationship of the magnitudes of the x components of T_1 and T_2?

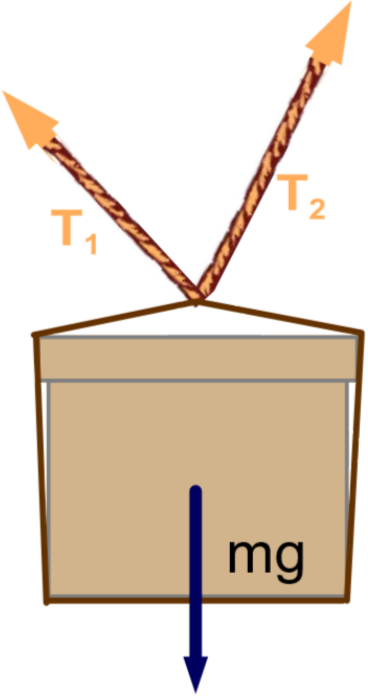
○ A $T_{1x} = T_{2x}$

○ B T_{1x} is greater than T_{2x}

○ C T_{1x} is less than T_{2x}

○ D $T_{1x} = T_{2x} = 0$

○ E I need help



https://www.njctl.org/video/?v=_PdzoT90NEM

Answer

45 In the case of two ropes holding up a stationary bucket, what is the relationship between mg and the magnitudes of the y components of T_1 and T_2?

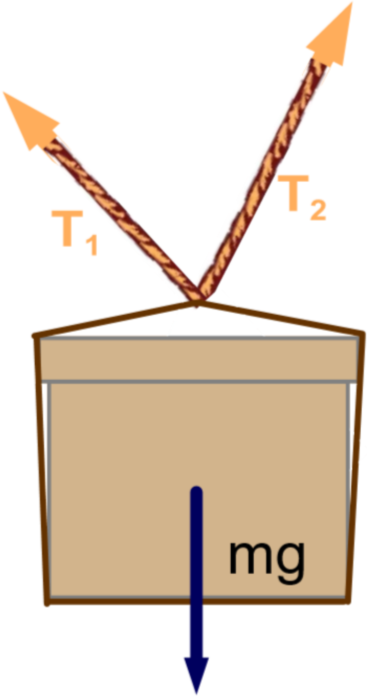
○ A $T_{1y} = T_{2y} + mg$

○ B $T_{1y} = T_{2y} - mg$

○ C $T_{1y} + T_{2y} = mg$

○ D $T_{1y} = T_{2y} = mg$

○ E I need help



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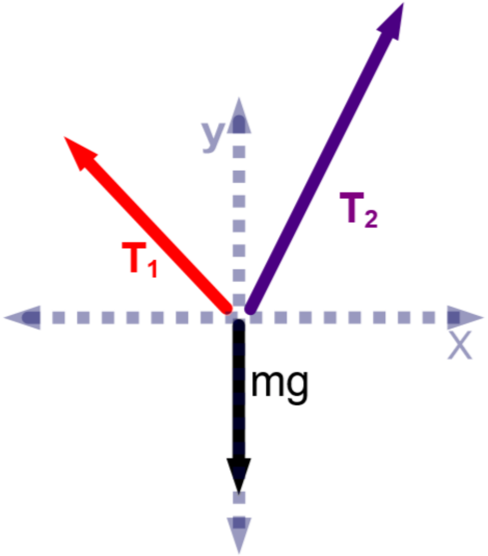
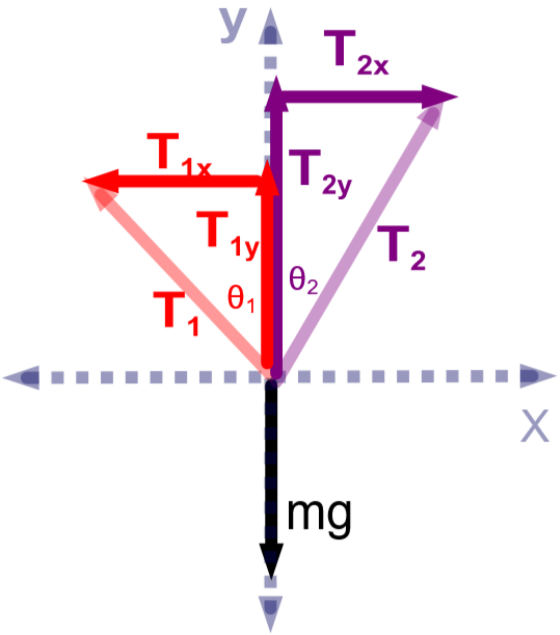
Answer

Tension Force

T_1 and T_2 will now be resolved along the x and y axes. mg is already just in the negative y axis direction.





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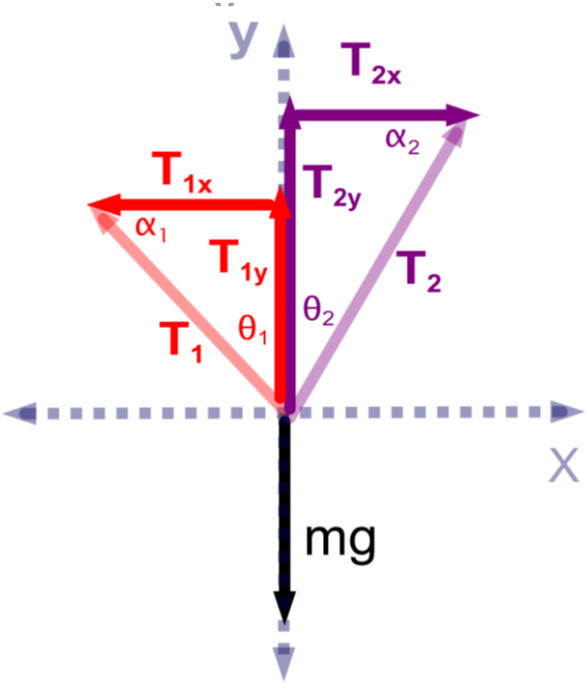
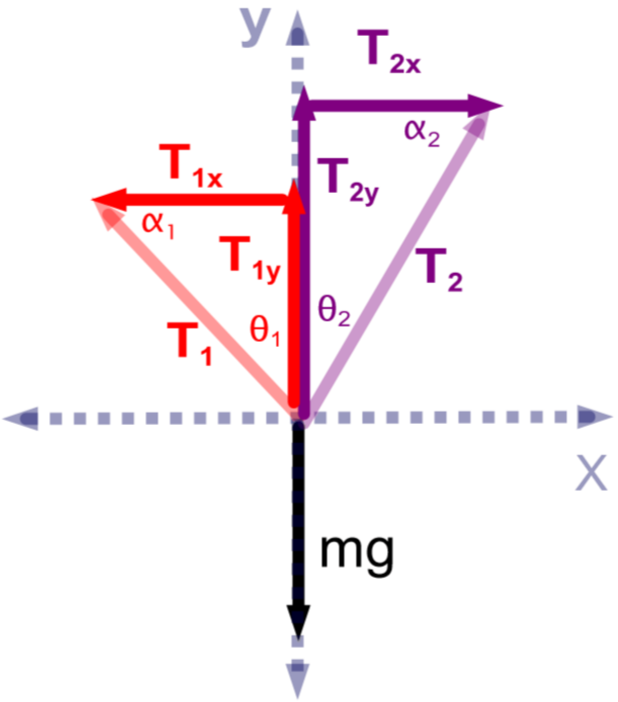
Tension Force

Be careful of how the problem is stated. Sometimes the angles α_1 and α_2 - the angles that the ropes make with the support platform (ceiling, for example) are given. They are complementary to the angles θ that are used here.



Tension Force



Time for a problem.

Calculate the tension in the two ropes if the first, T_1, is at an angle of 50^o from the vertical and the second, $T_{2,}$ is at an angle of 20^o from the vertical and they are supporting an 8.0 kg mass.

Note we are using θ_1 and θ_2 and assume two significant figures for each angle.

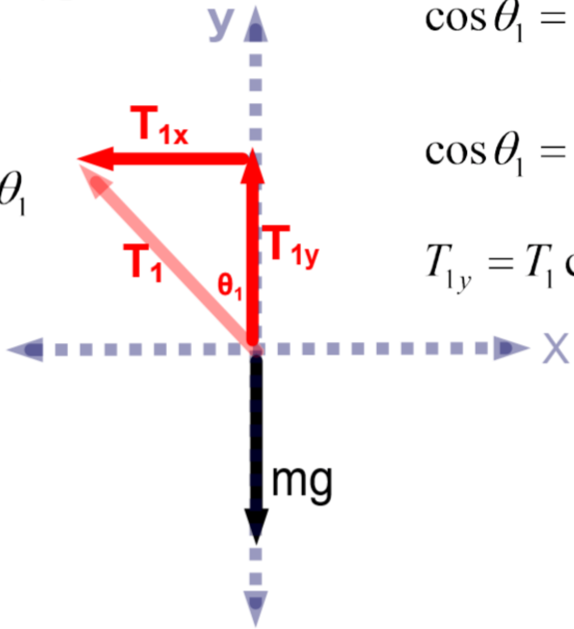
Tension Force

Resolve T $_1$ into its x and y components:

$$\sin\theta_1 = {}^{opp}\!/\!_{hyp}$$

$$\sin\theta_1 = \frac{T_{1x}}{T_1}$$

$$T_{1x} = T_1 \sin\theta_1$$



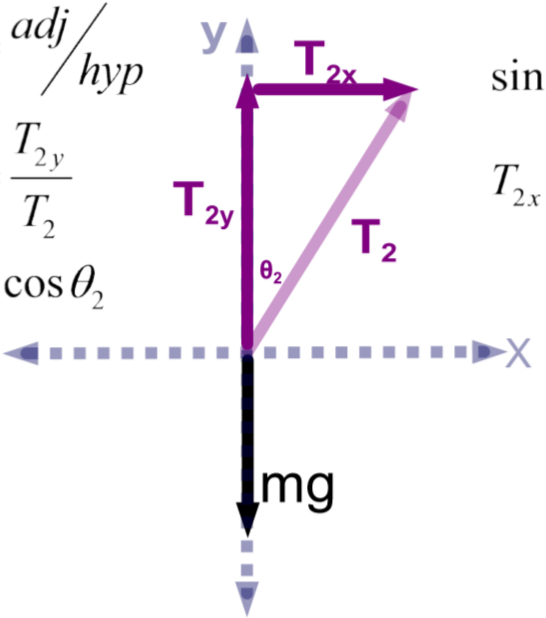
$$\cos\theta_1 = {}^{adj}\!/\!_{hyp}$$

$$\cos\theta_1 = \frac{T_{1y}}{T_1}$$

$$T_{1y} = T_1 \cos\theta_1$$

Tension Force

Resolve T_2 into its x and y components:



$$\cos\theta_2 = {}^{adj}\!/_{hyp}$$

$$\cos\theta_2 = \frac{T_{2y}}{T_2}$$

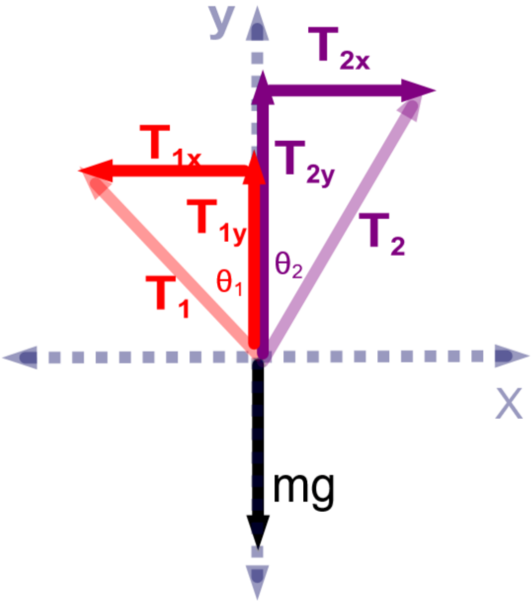
$$T_{2y} = T_2\cos\theta_2$$

$$\sin\theta_2 = {}^{opp}\!/_{hyp}$$

$$\sin\theta_2 = \frac{T_{2x}}{T_2}$$

$$T_{2x} = T_2\sin\theta_2$$

46 Given that the Tension in rope 1 is T_1 = 68 N, and $\theta_1 = 55^0$, find the x component of the Tension force.



○ A 41 N

○ B 56 N

○ C 74 N

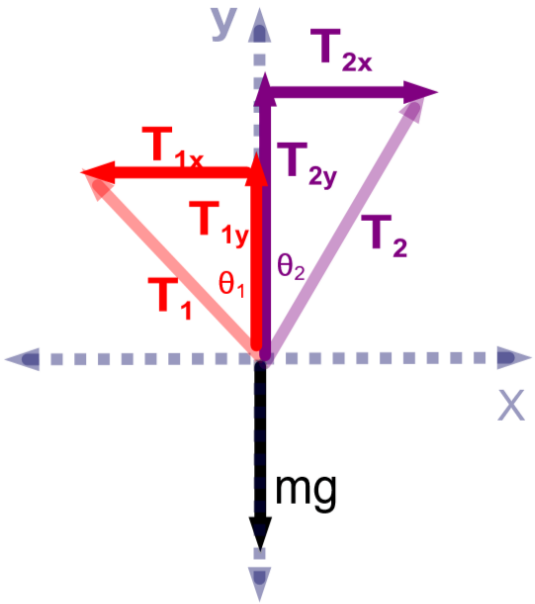
○ D 88 N

○ E I need help

https://www.njctl.org/video/?v=w5o_aB9wfwU

Answer

47 Given that the Tension in rope 2 is T_2 = 79 N, and $\theta_2 = 45^0$, find the x component of the Tension force.



○ A 44 N

○ B 56 N

○ C 77 N

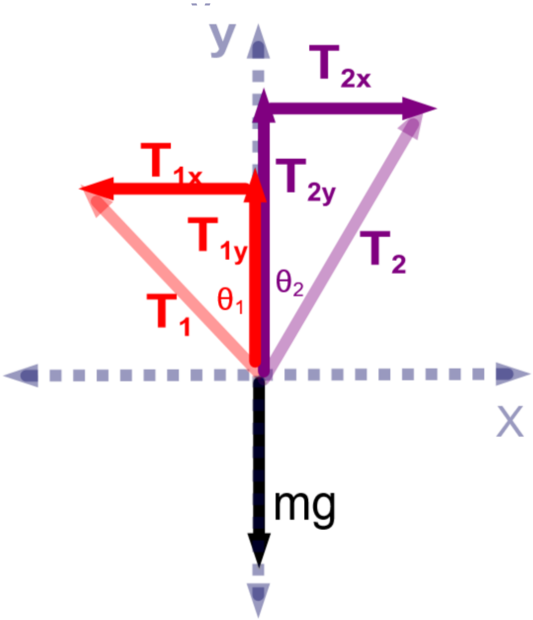
○ D 82 N

○ E I need help

Answer

https://www.njctl.org/video/?v=Wswnrve7R00

48 Given that the Tension in rope 1 is $T_1 = 68$ N, and $\theta_1 = 55^0$, find the y component of the Tension force.



○ A 23 N

○ B 39 N

○ C 44 N

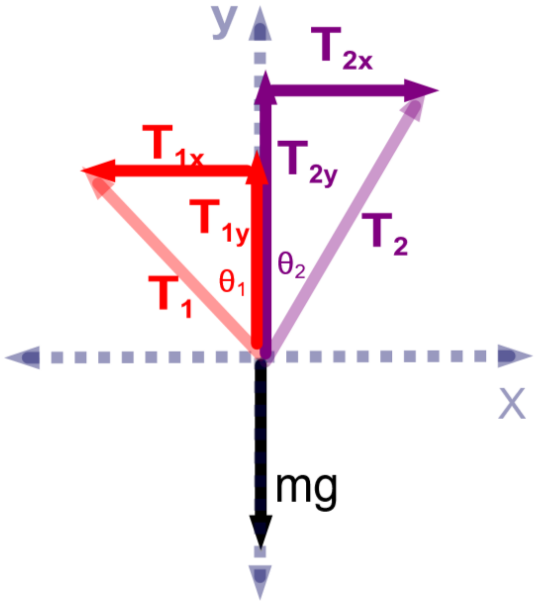
○ D 69 N

○ E I need help

Answer

https://www.njctl.org/video/?v=_XZhKk21xC0

49 Given that the Tension in rope 2 is T_2 = 79 N, and $\theta_2 = 45^0$, find the y component of the Tension force.



○ 38 N

○ 44 N

○ 51 N

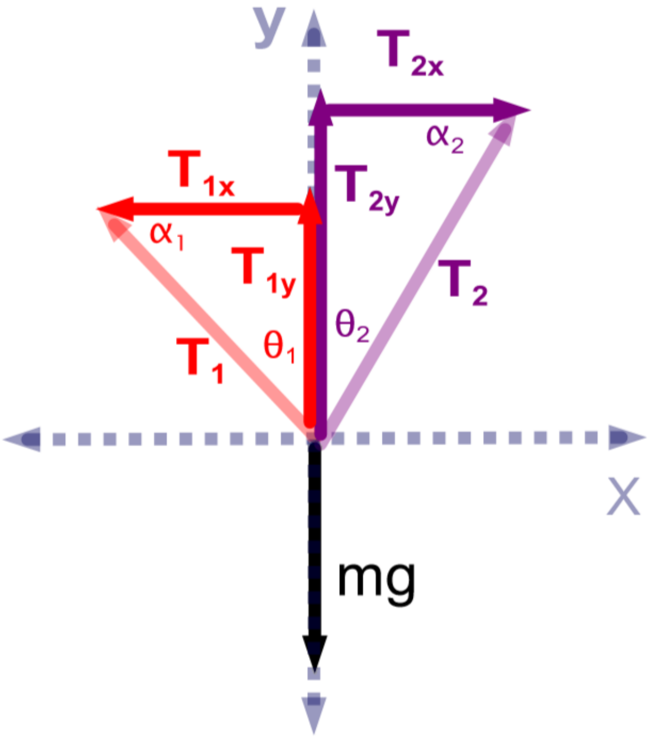
○ 56 N

○ I need help

https://www.njctl.org/video/?v=2_1gYm4cRQ8

Answer

Tension Force



It's now time to take the resolved vectors, substitute in the given values and solve the simultaneous equations:

$\theta_1 = 50^o$
$\theta_2 = 20^o$
$m = 8.0$ kg

$$T_{1x} = T_1 \sin\theta_1 \quad T_{2x} = T_2 \sin\theta_2$$

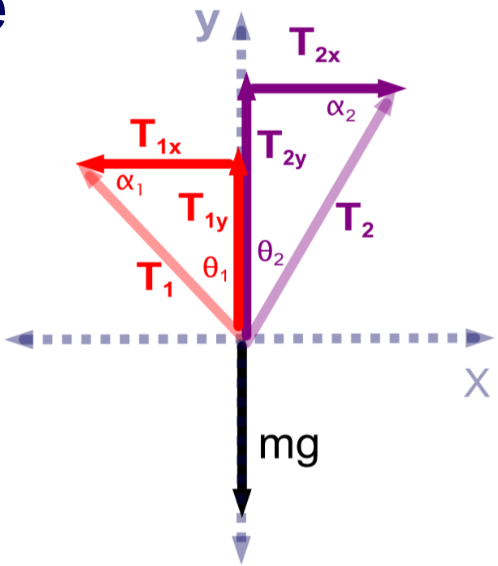
$$T_{1y} = T_1 \cos\theta_1 \quad T_{2y} = T_2 \cos\theta_2$$

$$T_{1y} + T_{2y} = mg$$

$$T_{1x} = T_{2x}$$

https://www.njctl.org/video/?v=1ye_jJtPiWg

Tension Force



$$T_{1x} = T_1 \sin\theta_1 = T_1 \sin(50^0) = .77T_1$$

$$T_{1y} = T_1 \cos\theta_1 = T_1 \cos(50^0) = .64T_1$$

$$T_{2x} = T_2 \sin\theta_2 = T_2 \sin(20^0) = .34T_2$$

$$T_{2y} = T_2 \cos\theta_2 = T_2 \cos(20^0) = .94T_2$$

$$mg = (8.0)(9.8) = 78$$

Recall that the signs are already taken into account with the following two equations, so just substitute in the above values. *That's the advantage of a FBD and not worrying about the signs of the sine and cosine functions.*

$$T_{1y} + T_{2y} = mg \qquad .64T_1 + .94T_2 = 78$$

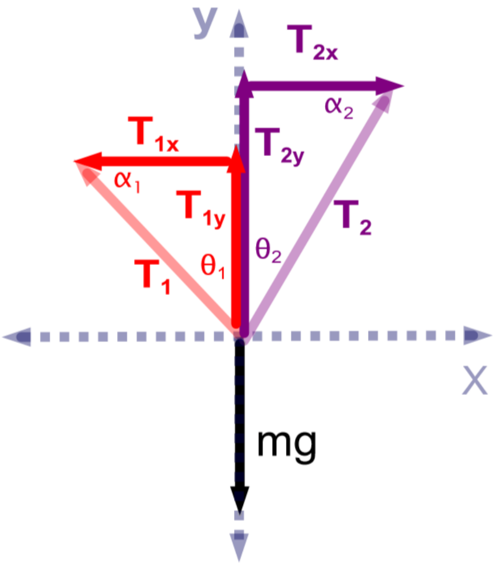
$$-T_{1x} + T_{2x} = 0$$

$$T_{1x} = T_{2x} \qquad .77T_1 = .34T_2$$

Tension Force

Equation 1: $.64T_1 + .94T_2 = 78$

Equation 2: $.77T_1 = .34T_2$



We now have two simultaneous equations with two variables. Solve Equation 2 for T_1 and then substitute T_1 into Equation 1 and solve for T_2.

$$T_1 = \frac{.34}{.77} T_2 = .44T_2$$

$$.64(.44T_2) + .94T_2 = 78$$

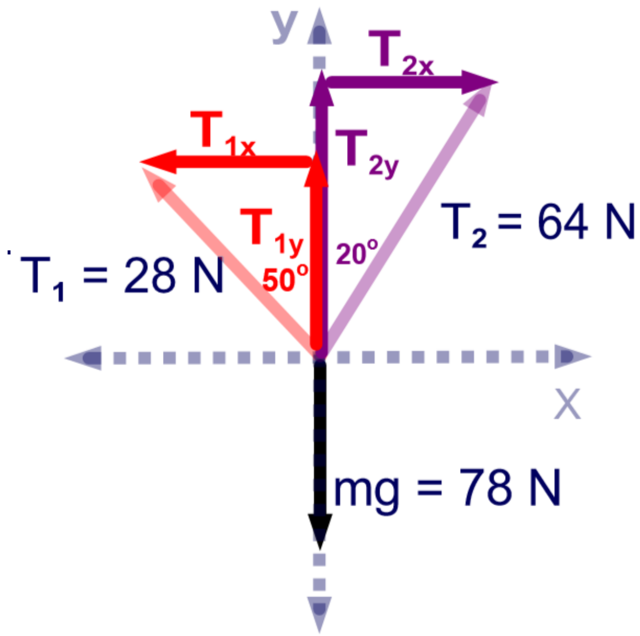
$$T_2 = 64N$$

Now substitute T_2 into Equation 2 and solve for T_1.

$$.77T_1 = .34(64)$$

$$T_1 = 28N$$

Tension Force

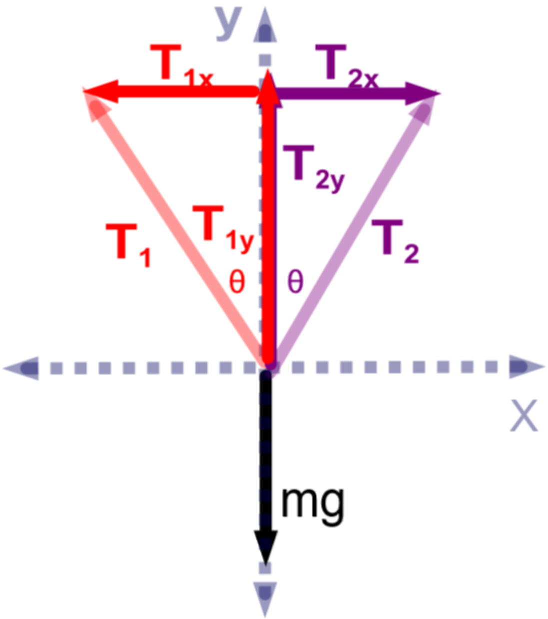


T_2 is greater than T_1, which means that it is picking up a greater "load." A way to determine this without the mathematics is to note that T_2 is more vertical than T_1.

So if two people are trying to lift a load - the stronger person should take the more vertical rope!

Also the sum of the magnitudes of T_1 and T_2 are greater than the weight of the box.

Tension Force



Take a limiting case where $\theta_1 = \theta_2 = \theta$. The ropes make equal angles with the vertical.

$$T_{1x} = T_1 \sin\theta_1 = T_1 \sin\theta$$

$$T_{2x} = T_2 \sin\theta_2 = T_2 \sin\theta$$

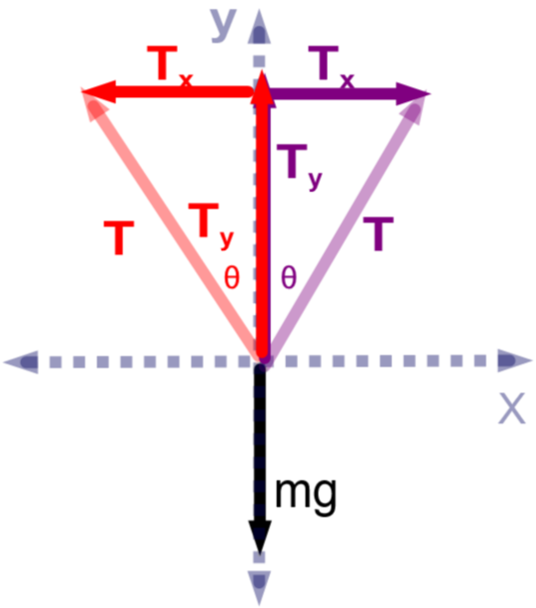
Since $T_{1x} = T_{2x}$

$$T_1 \sin\theta = T_2 \sin\theta$$

$$T_1 = T_2 = T$$

Each rope has the same tension - the load is shared equally.

Tension Force



Now, examine the forces in the y direction.

$$T_{1y} = T_1 \cos\theta_1 = T\cos\theta$$

$$T_{2y} = T_2 \cos\theta_2 = T\cos\theta$$

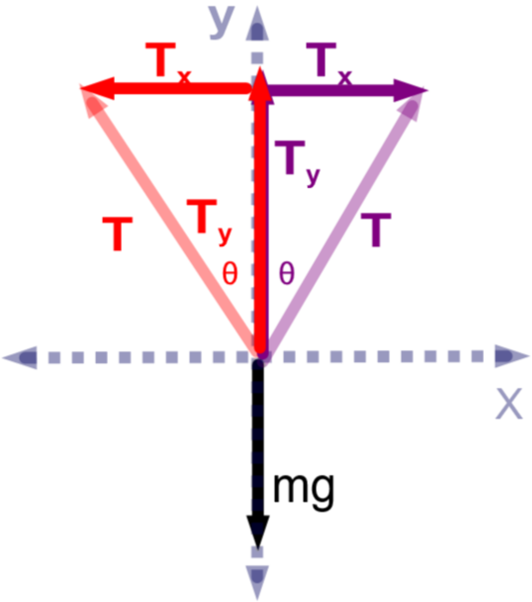
Since $T_{1y} + T_{2y} = mg$

$$T\cos\theta + T\cos\theta = mg$$

$$T = \frac{mg}{2\cos\theta}$$

Let's take a limiting case again (physicists love doing this) - what happens as the support wires get more horizontal (θ approaches 90^0)?

Tension Force



$$T = \frac{mg}{2\cos\theta}$$

As θ approaches 90^0 (the ropes become more horizontal), the Tension required to support the box approaches infinity.

Can you think of an example of this effect (and you can't use the traffic light one again)?

Tension Force

http://commons.wikimedia.org/wiki/File:Shazand_Thermal_power_plant.JPG

Electrical power transmission lines.

This helps explain why the lines sag - the force required to maintain a horizontal position would exceed the strength of the lines - and external wires are wrapped around them to add more support.

50 A lamp of mass m is suspended from two ropes of unequal length as shown below. Which of the following is true about the tensions T_1 and T_2 in the ropes?

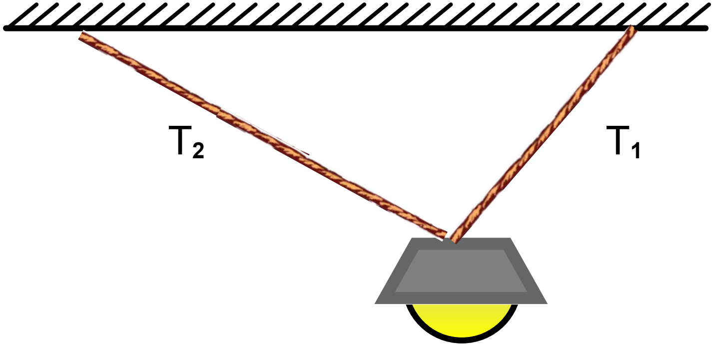
○ A T_1 is less than T_2

○ B $T_1 = T_2$

○ C T_1 is greater than T_2

○ D $T_1 + T_2 = mg$

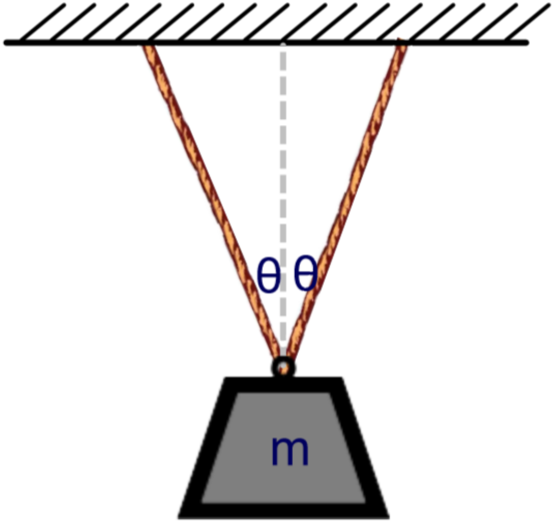
○ I need help



Answer

https://www.njctl.org/video/?v=xtNvY4q5AGE

51 A mass m is suspended from two massless strings of an equal length as shown below. The tension force in each string is:



○ A mg/(2cosθ)

○ B 2 mgcosθ

○ C mgcosθ

○ D mg/cosθ

○ E I need help

Answer

https://www.njctl.org/video/?v=OAloKqfZ-QA