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NEW JERSEY CENTER
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AP Physics 1

Circular Motion

2017-07-19
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Topics of Uniform Circular Motion (UCM)

Click on the topic to go to that section

Kinematics of UCM

Demo

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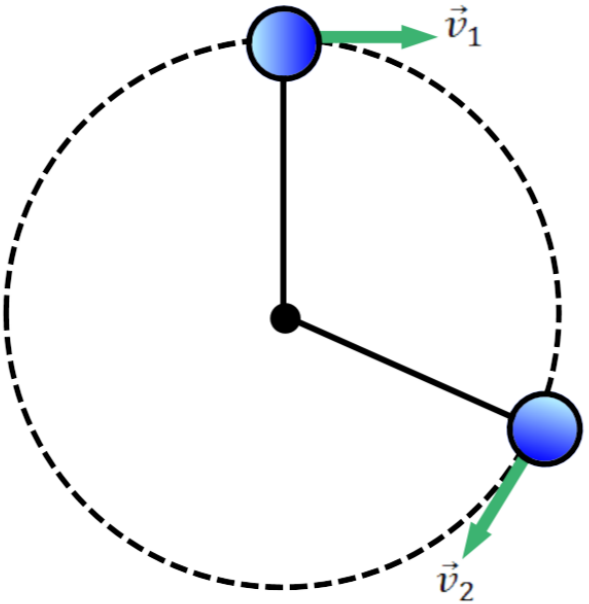
Kinematics of Uniform Circular Motion

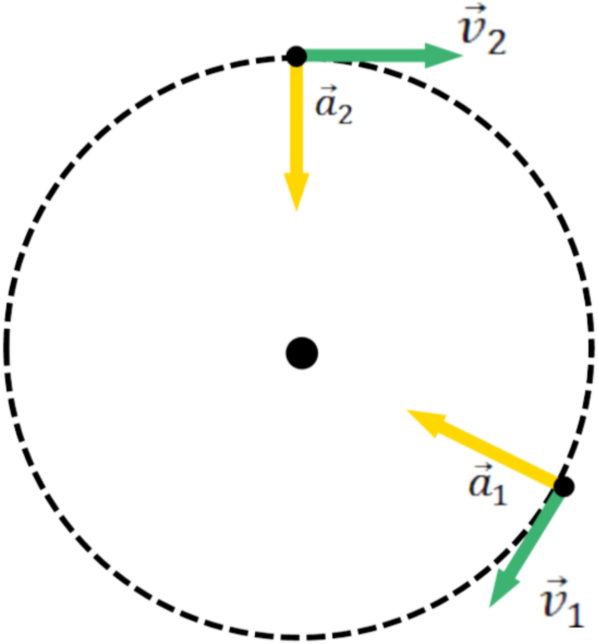
Uniform circular motion: motion in a circle of constant radius at constant speed

Instantaneous velocity is always tangent to circle.



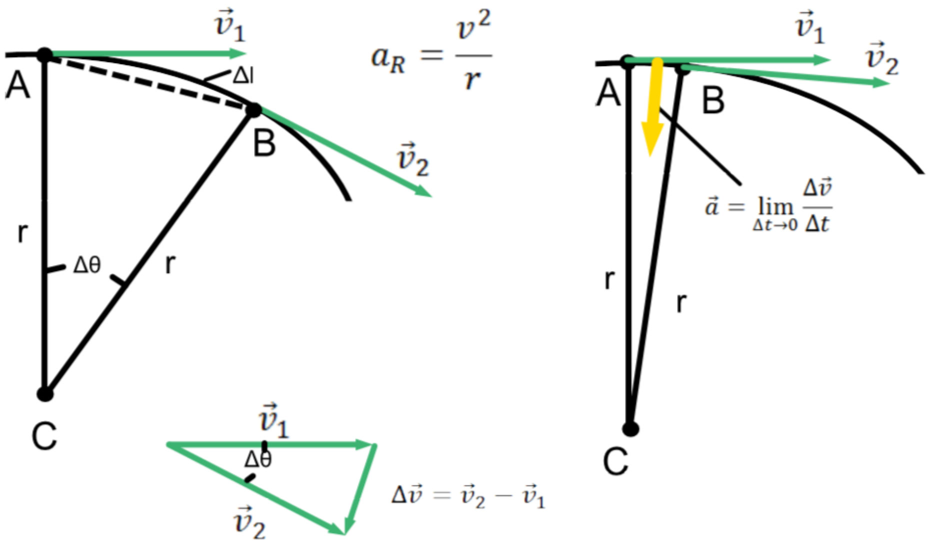
Kinematics of Uniform Circular Motion

This acceleration is called the centripetal, or radial, acceleration, and it points towards the center of the circle.



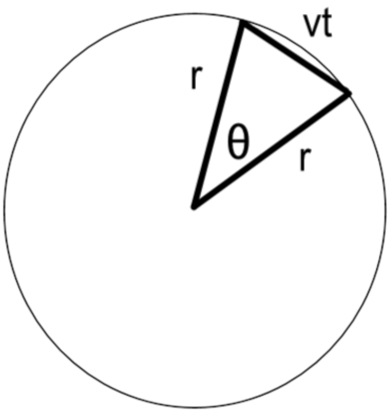
Kinematics of Uniform Circular Motion

Looking at the change in velocity in the limit that the time interval becomes infinitesimally small, we see that we get two similar triangles.

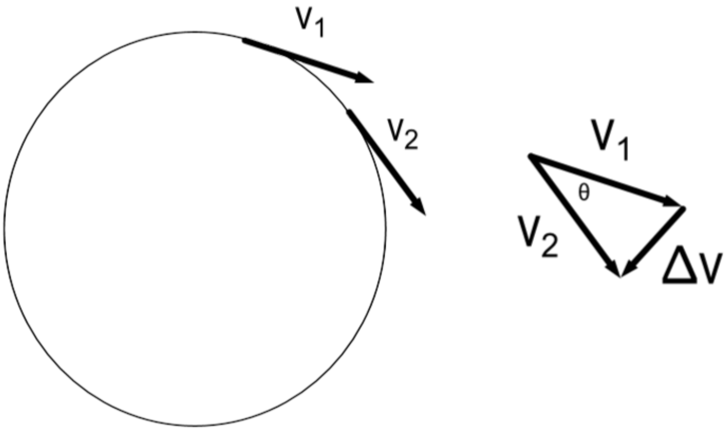


Kinematics of Uniform Circular Motion

If displacement is equal to velocity multiplied by time, then vt is the displacement covered in time t.



During that same time, velocity changed by an amount, Δv.



Kinematics of Uniform Circular Motion

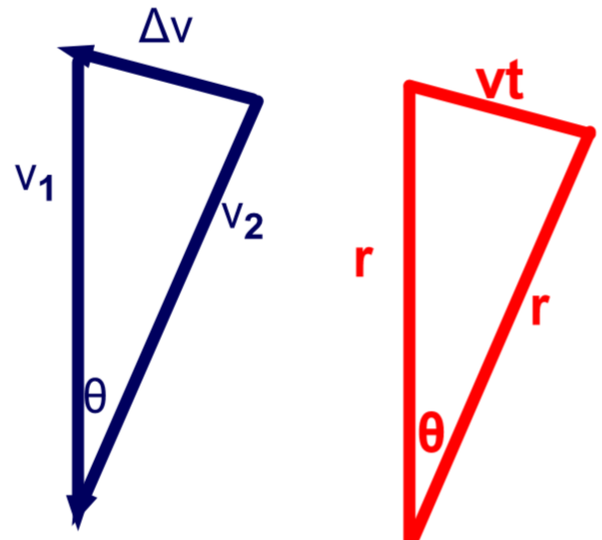
These are similar triangles because the angles are all congruent, so the sides must be in proportion.

$$\frac{\Delta v}{v} = \frac{vt}{r}$$

$$\frac{\Delta v}{t} = \frac{v^2}{r}$$

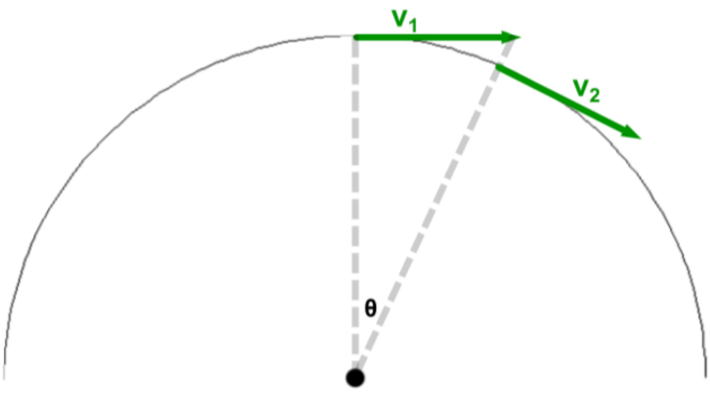
$$a = \frac{v^2}{r}$$

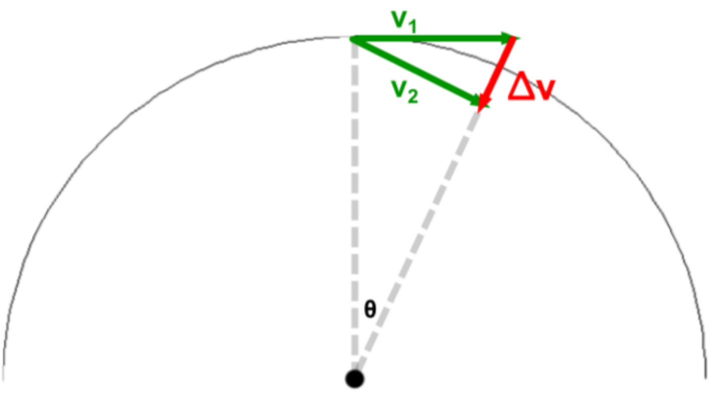
That's the magnitude of the acceleration.



Kinematics of Uniform Circular Motion

Transpose v_2 to see the vector addition.





The change in velocity, Δv, shows the direction of the acceleration.

In the picture, you can see Δv points towards the center of the circle.

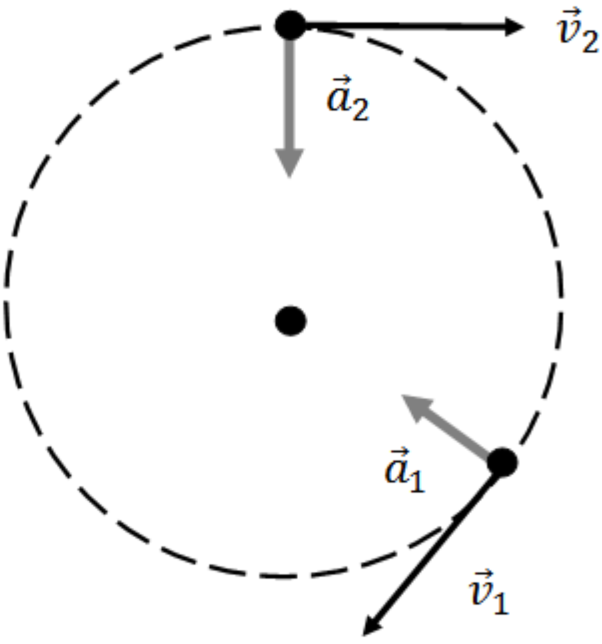
Kinematics of Uniform Circular Motion

This acceleration is called the centripetal, or radial, acceleration.

Its direction is towards the center of the circle.

Its magnitude is given by

$$a = \frac{v^2}{r}$$



1 Is it possible for an object moving with a constant speed to accelerate? Explain.

- A No, if the speed is constant then the acceleration is equal to zero.
- B No, an object can accelerate only if there is a net force acting on it.
- C Yes, although the speed is constant, the direction of the velocity can be changing.
- D Yes, if an object is moving it is experiencing acceleration.
- E I need help

Answer

2 Consider a particle moving with constant speed such that its acceleration of constant magnitude is always perpendicular to its velocity.

- A It is moving in a straight line.
- B It is moving in a circle.
- C It is moving in a parabola.
- D None of the above is definitely true all of the time.
- E I need help

Answer

3 An object moves in a circular path at a constant speed. Compare the direction of the object's velocity and acceleration vectors.

- A Both vectors point in the same direction.
- B The vectors point in opposite directions.
- C The vectors are perpendicular.
- D The question is meaningless, since the acceleration is zero.
- E I need help

Answer

4 Two cars go around the same circular track at the same speed. The first car is closer to the inside of the circle. Which of the following is true about their centripetal acceleration?

- A Both cars have the same centripetal motion since they both have the same speed.
- B The centripetal acceleration of the first car is greater since its radius is smaller.
- C The centripetal acceleration of the second car is greater since its radius is larger.
- D The centripetal acceleration of the first car is less since its radius is smaller.
- E I need help

Answer

Period, Frequency, and Rotational Velocity

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Period

The time it takes for an object to complete one trip around a circular pathis called its Period.

The symbol for Period is "T"

Periods are measured in units of time; we will usually use seconds (s).

Often we are given the time (t) it takes for an object to make a number of trips (n) around a circular path. In that case,

$$T = \frac{t}{n}$$

Frequency

The number of revolutions that an object completes in a given amount of time is called the frequency of its motion.

The symbol for frequency is "f"

Frequencies are measured in units of revolutions per unit time; we will usually use 1/seconds (s^{-1}). Another name for s^{-1} is Hertz (Hz). Frequency can also be measured in revolutions per minute (rpm), etc.

Often we are given the time (t) it takes for an object to make a number of revolutions (n).
In that case,

$$f = \frac{n}{t}$$

Period and Frequency

Period

$$T = \frac{t}{n}$$

Frequency

$$f = \frac{n}{t}$$

These two equations look similar.
In fact, they are exactly opposite one another.

$$\frac{t}{n} \quad \frac{n}{t}$$

Another way to say this is that they are inverses.

Period and Frequency

We can relate them mathematically:

Period is the inverse of Frequency	$T = \frac{1}{f}$
Frequency is the inverse of Period	$f = \frac{1}{T}$

Rotational Velocity

In kinematics, we defined the speed of an object as

$$s = \frac{d}{t}$$

For an object moving in a circle, instead of using t (time), we will measure the speed with T (period), the time it takes to travel around a circle.

To find the speed, then, we need to know the distance around the circle.

Another name for the distance around a circle is the circumference.

Rotational Velocity

Each trip around a circle, an object travels a length equal to the circle's circumference.

The circumference of a circle is given by $C = 2\pi r$

The time it takes to go around once is the period T

And the object's speed is given by $s = \frac{d}{t}$

So the magnitude of its velocity must be:

$$v = \frac{2\pi r}{T}$$

Rotational Velocity

A velocity must have a magnitude and a direction.

The magnitude of an object's instantaneous velocity is its speed. So for an object in uniform circular motion, the magnitude of its velocity is:

$$v = \frac{2\pi r}{T}$$

If an object is in uniform circular motion, the direction of its velocity is always changing!

We say the velocity is tangent to its circular motion.

*Rotational Velocity

Since $f = \frac{1}{T}$, we can also determine the velocity of an object in uniform circular motion by the radius and frequency of its motion.

$$v = \frac{2\pi r}{T}$$ *and* $$f = \frac{1}{T}$$ *so* $$v = 2\pi r f$$

Of course the direction of its velocity is still tangent to its circular motion.

5 A girl whirls a toy at the end of a string around her head. The string makes one complete revolution every second. She keeps the radius constant but increases the speed so that the string makes two complete revolutions per second. What happens to the centripetal acceleration?

- A The centripetal acceleration remains the same.
- B The centripetal acceleration doubles.
- C The centripetal acceleration quadruples.
- D The centripetal acceleration is cut to half.
- E I need help

Answer

6 A ball is swung in a circle. The frequency of the ball is doubled. By what factor does the period change?

- A It remains the same.
- B It is cut to one half.
- C It doubles.
- D It is cut to one fourth.
- E I need help

Answer

Dynamics of UCM

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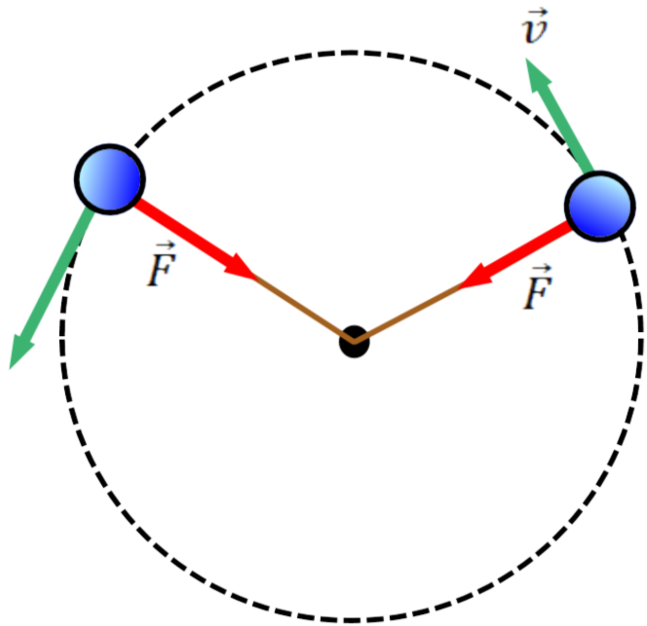
Dynamics of Uniform Circular Motion

For an object to be in uniform circular motion, there must be a net force acting on it.

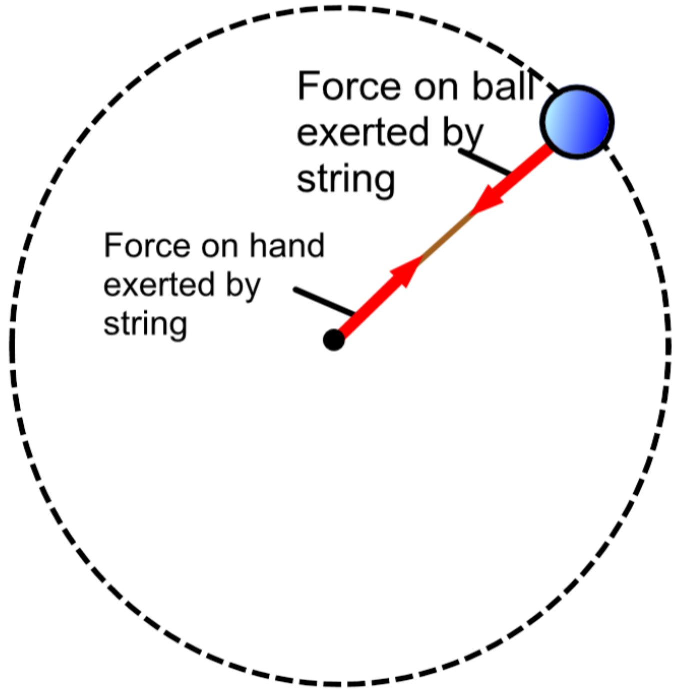
We already know the acceleration, so we can write the force:

$$\Sigma F_R = m a_R = m \frac{v^2}{r}$$



Dynamics of Uniform Circular Motion

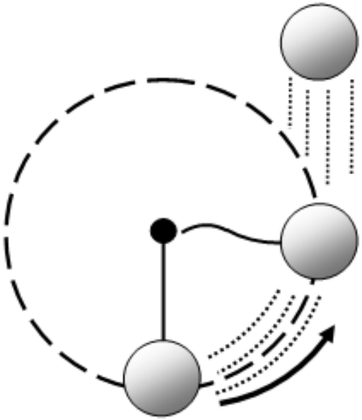
We can see that the force must be inward by thinking about a ball on a string:



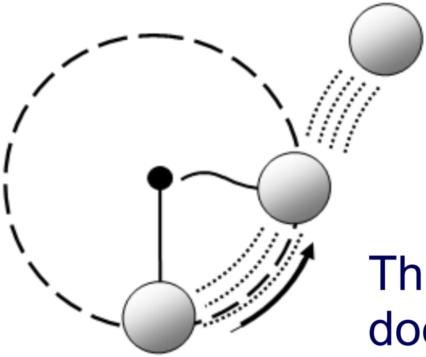
Dynamics of Uniform Circular Motion

There is no centrifugal force pointing outward; what happens is that the natural tendency of the object to move in a straight line must be overcome.

If the centripetal force vanishes, the object flies off tangent to the circle.

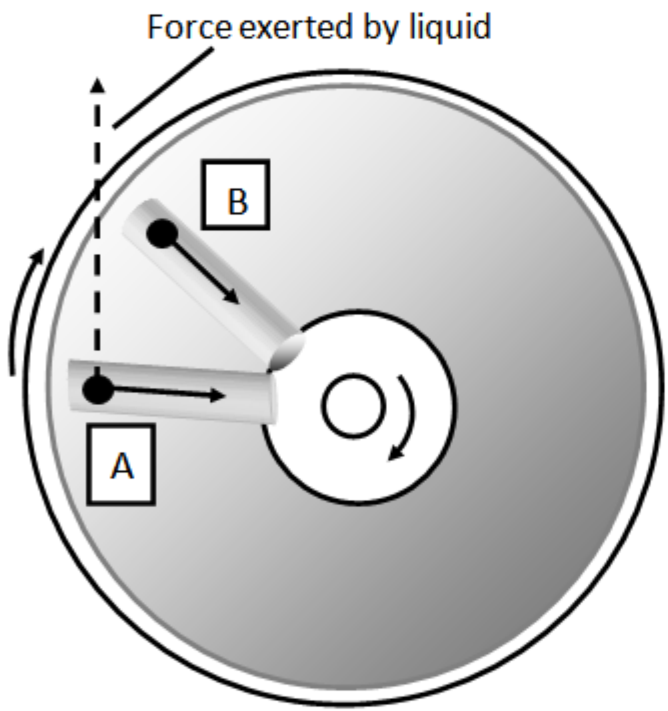


This happens.



This does NOT happen.

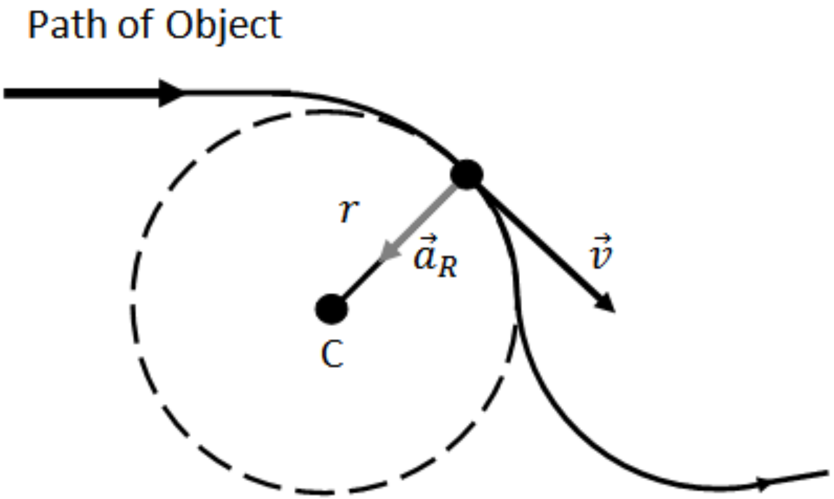
Centrifugation



A centrifuge works by spinning very fast. This means there must be a very large centripetal force. The object at A would go in a straight line but for this force; as it is, it winds up at B.

Curved Paths

This concept can be used for an object moving along any curved path , as a small segment of the path will be approximately circular.



7 When an object experiences uniform circular motion, the direction of the net force is:

- A in the same direction as the motion of the object.
- B in the opposite direction of the motion of the object.
- C is directed toward the center of the circular path.
- D is directed away from the center of the circular path.
- E I need help

Answer

8 A boy whirls a toy at the end of a string around his head. The string makes one complete revolution every second. He keeps the radius constant but decreases the speed so that the string makes one revolution every two seconds. What happens to the tension in the string?

- A The tension remains the same.
- B The tension doubles.
- C The tension is cut to half.
- D The tension is cut to one fourth.
- E I need help

Answer

9 (Multi-correct Directions: For each of the following, two of the suggested answers will be correct. Select the best two choices to earn credit. No partial credit will be earned if only one correct choice is selected.)

An object is moving in uniform circular motion with a mass m, a speed v, and a radius r. Which of the following will quadruple the centripetal force on the object?

- ☐ A Doubling the speed.
- ☐ B Cutting the speed to one half.
- ☐ C Cutting the radius to one half.
- ☐ D Cutting the radius to one fourth.
- ☐ I need help

Answer

Vertical UCM

Demo

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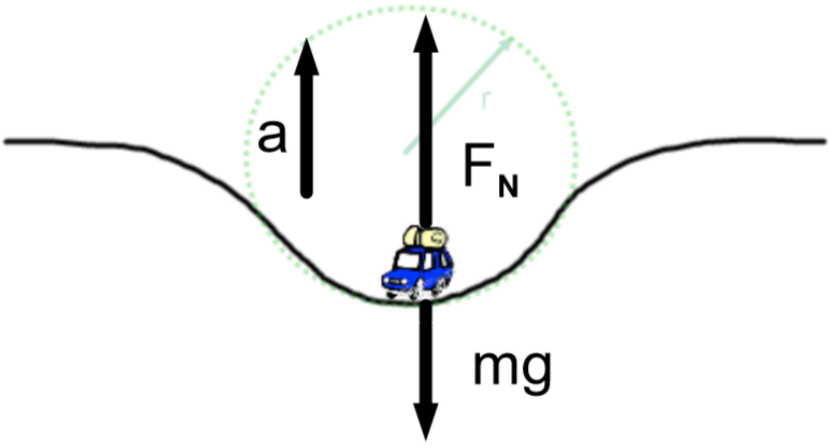
Car on a hilly road

When a car goes though a dip, we can consider it to be in circular motion. Its acceleration is towards the center of the circle, which is up. We can use a free body diagram and Newton's second law to derive an equation for the normal force on the car.

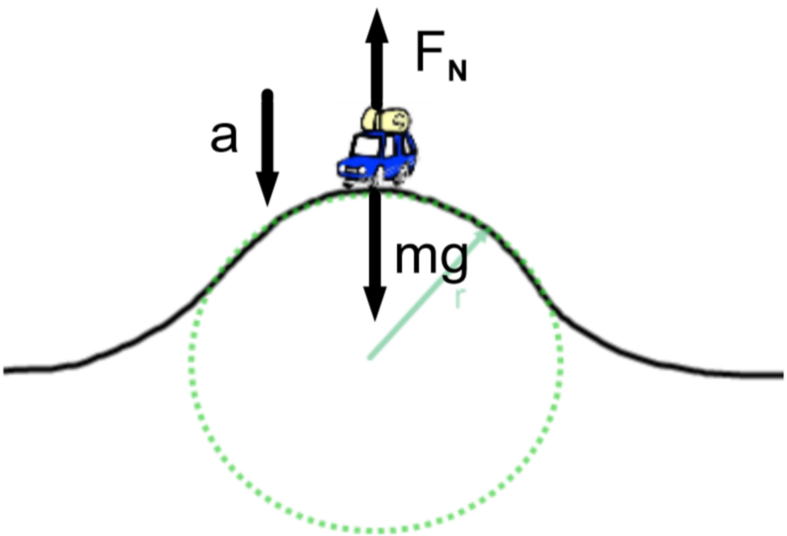


$$\sum F = ma$$

$$F_N - mg = \frac{mv^2}{r}$$

Car on a hilly road

When a car goes over a hill, we can consider it to be in circular motion. Its acceleration is towards the center of the circle, which is down. We can use a free body diagram and Newton's second law to derive an equation for the normal force on the car.

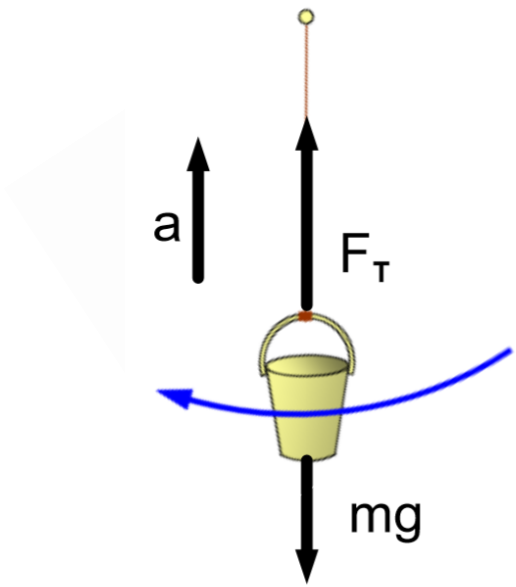


$$\sum F = ma$$

$$F_N - mg = -\frac{mv^2}{r}$$

Buckets and Rollercoasters

A bucket on a string moving in a vertical circle is also in circular motion. When it is at the bottom of the circle, it is in the same situation at a car going through a dip.

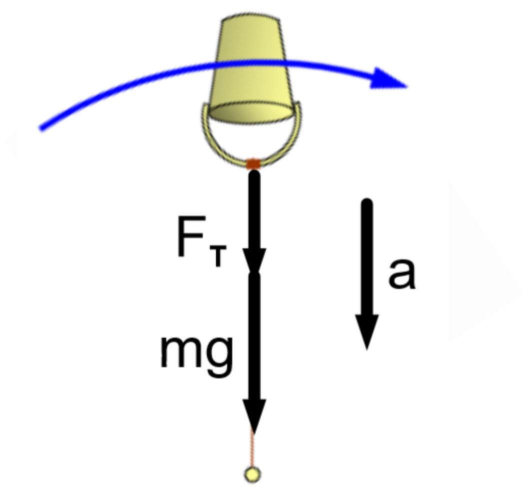


$$\sum F = ma$$

$$F_T - mg = \frac{mv^2}{r}$$

Buckets and Rollercoasters

A bucket on a string moving in a vertical circle is also in circular motion. When it is at the top of the circle, there is no force upward. The tension and weight are both down.

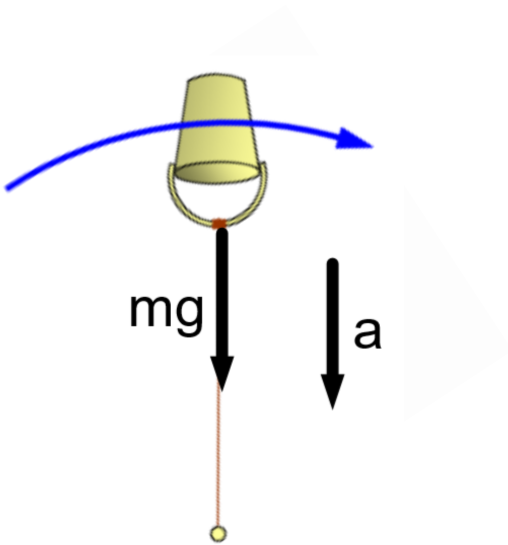


$$\sum F = ma$$

$$F_T + mg = \frac{mv^2}{r}$$

Buckets and Rollercoasters

The minimum velocity for a bucket to make it around the circle is achieved when the tension in the string becomes zero at the top of the circle.



$$\sum F = ma$$

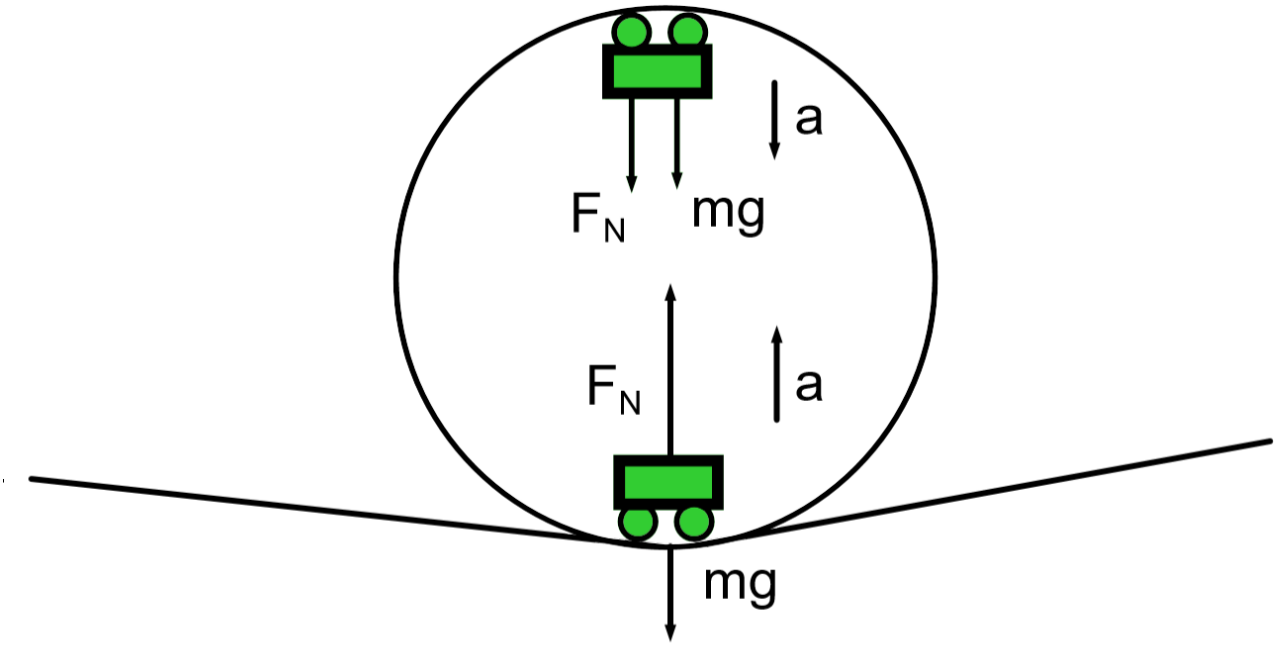
$$mg = \frac{mv^2}{r}$$

$$g = \frac{v^2}{r}$$

$$v = \sqrt{gr}$$

Buckets and Rollercoasters

A roller coaster car going around a loop works exactly like a bucket on a string. The only difference is that instead of tension, there is a normal force exerted on the car.



10 A roller coaster car is on a track that forms a circular loop in the vertical plane. If the car is to just maintain contact with track at the top of the loop, what is the minimum value for its centripetal acceleration at this point?

- A g downward
- B 0.5g downward
- C g upward
- D 2g upward
- E I need help

Answer

11 A roller coaster car (mass = M) is on a track that forms a circular loop (radius = r) in the vertical plane. If the car is to just maintain contact with the track at the top of the loop, what is the minimum value for its speed at that point?

- A gr
- B $\sqrt{gr}$
- C $\sqrt{2gr}$
- D $\sqrt{\frac{gr}{2}}$
- E I need help

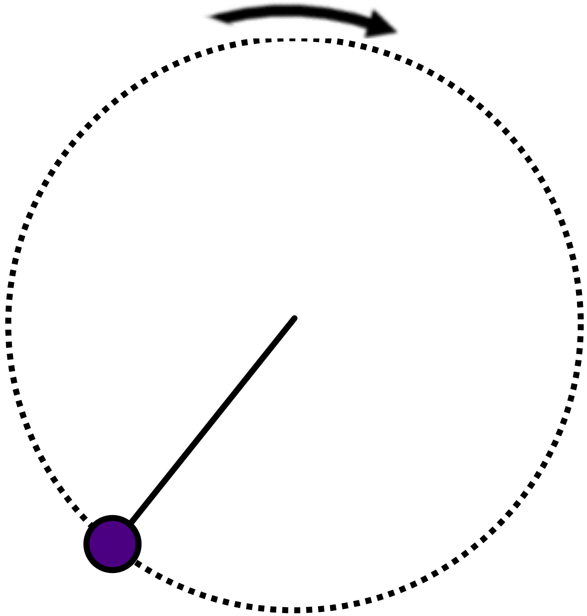
Answer

12 A pilot executes a vertical dive then follows a semi-circular arc until it is going straight up. Just as the plane is at its lowest point, the force of the seat on him is:

- A less than mg and pointing up.
- B less than mg and pointing down.
- C more than mg and pointing up.
- D more than mg and pointing down.
- E I need help

Answer

(Short answer) A ball is whirled on a string as shown in the diagram. The string breaks at the point shown. Draw the path the ball takes and explain why this happens.



Answer

Horizontal UCM

Demo

Lab

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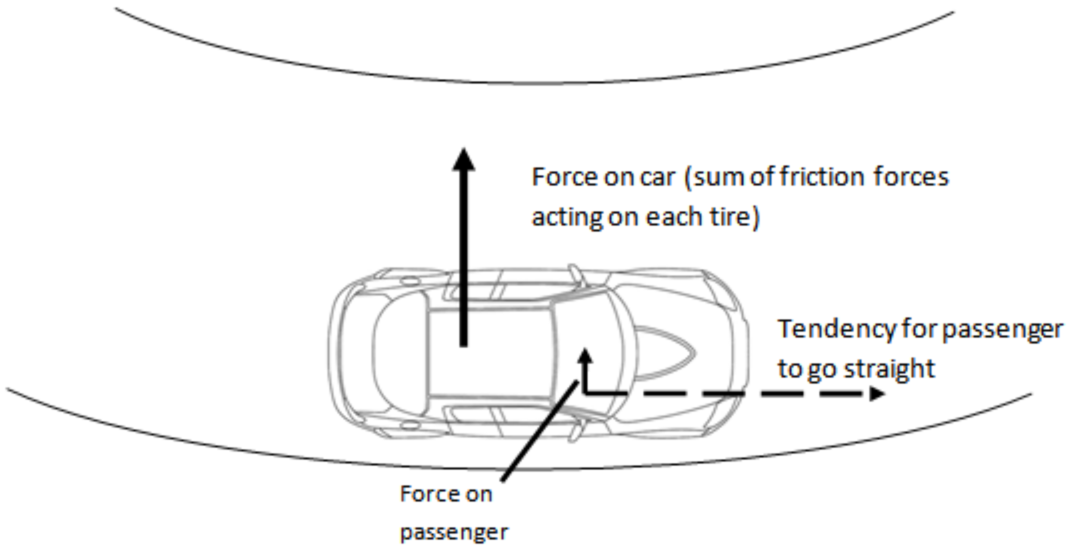
Banked and Unbanked Curves

When a car goes around a curve, there must be a net force towards the center of the circle of which the curve is an arc. If the road is flat, that force is supplied byfriction.



Banked and Unbanked Curves

If the frictional force is insufficient, the car will tend to move more nearly in a straight line, as the skid marks show.

Banked and Unbanked Curves

As long as the tires do not slip, the friction is static.

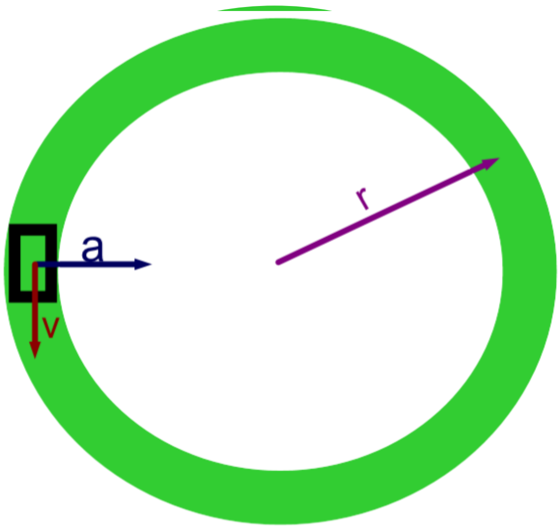
If the tires do start to slip, the friction is kinetic, which is bad in two ways:

1. The kinetic frictional force is smaller than the static.

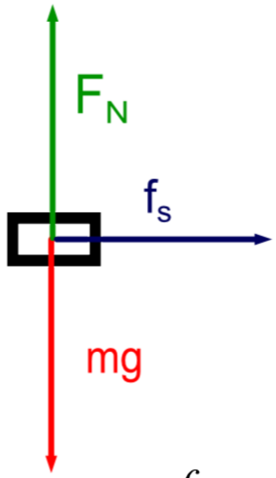
2. The static frictional force can point towards the center of the circle, but the kinetic frictional force opposes the direction of motion, making it very difficult to regain control of the car and continue around the curve.

Unbanked Curves

Top View



Front View
(the car heading towards you)



$$f_s = ma$$

$$F_N - mg = 0$$

$$F_N = mg$$

$$\mu mg = \frac{mv^2}{r}$$

$$\mu g = \frac{v^2}{r}$$

13 A car goes around a curve of radius r at a constant speed v. Then it goes around the same curve at half of the original speed. What is the centripetal force on the car as it goes around the curve for the second time, compared to the first time?

- A twice as big
- B four times as big
- C half as big
- D one-fourth as big
- E I need help

Answer

14 The top speed a car can go around an unbanked curve safely (without slipping) depends on all of the following except:

- A The coefficient of friction.
- B The mass of the car.
- C The radius of the curve.
- D The acceleration due to gravity.
- E I need help

Answer

15 You are sitting in the passenger seat of a car going a round a turn. Which of the following is responsible for keeping you moving in a circle?

- A The gravitational force.
- B The outward force of the car seat and seat belt.
- C The inward force of the car seat and seat belt.
- D The centrifugal force.
- E I need help

Answer

Banked Curves

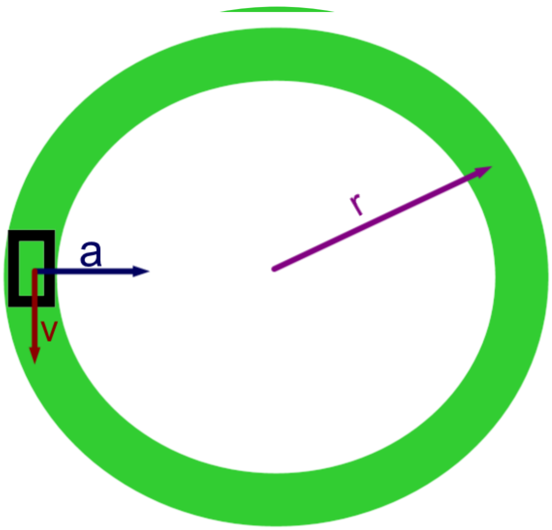
Banking curves can help keep cars from skidding.

In fact, for every banked curve, there is one speed where the entire centripetal force is supplied by the horizontal component of the normal force, and no friction is required.

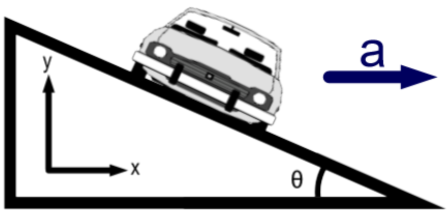
Let's figure out what that speed is for a given angle and radius of curvature.

Banked Curves

Top View



Front View
(the car heading towards you)

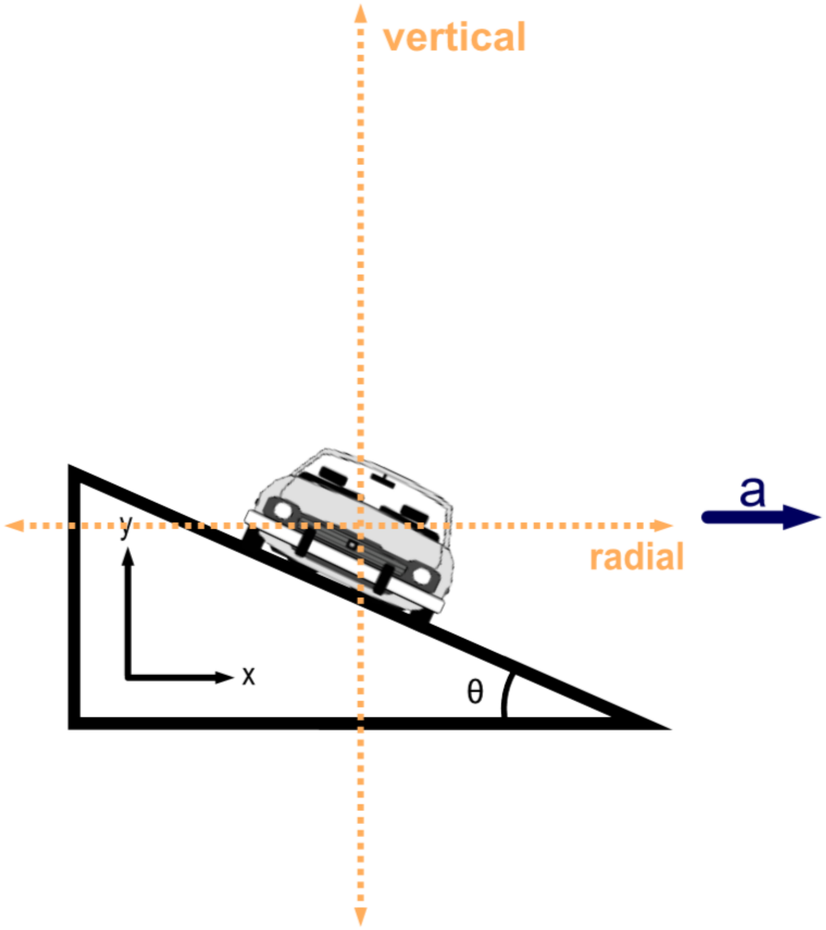


We know the direction of our acceleration, so now we have to create axes.

Note that these axes will be different than for an Inclined Plane, because the car is not expected to slide down the plane.

Instead, it must have a horizontal acceleration to go in a horizontal circle.

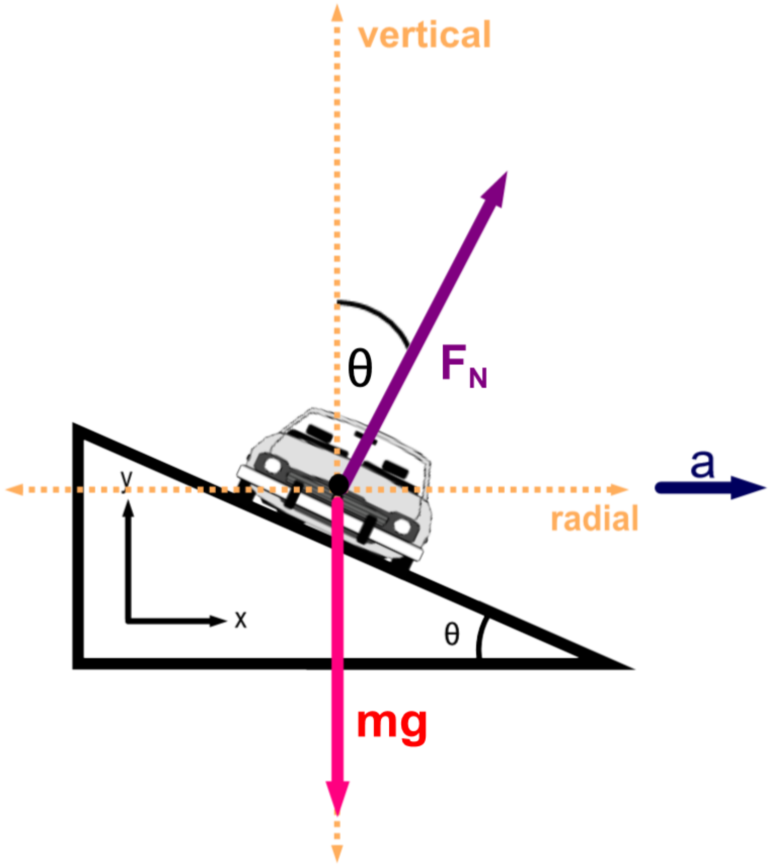
Banked Curves



First, do the free body diagram.

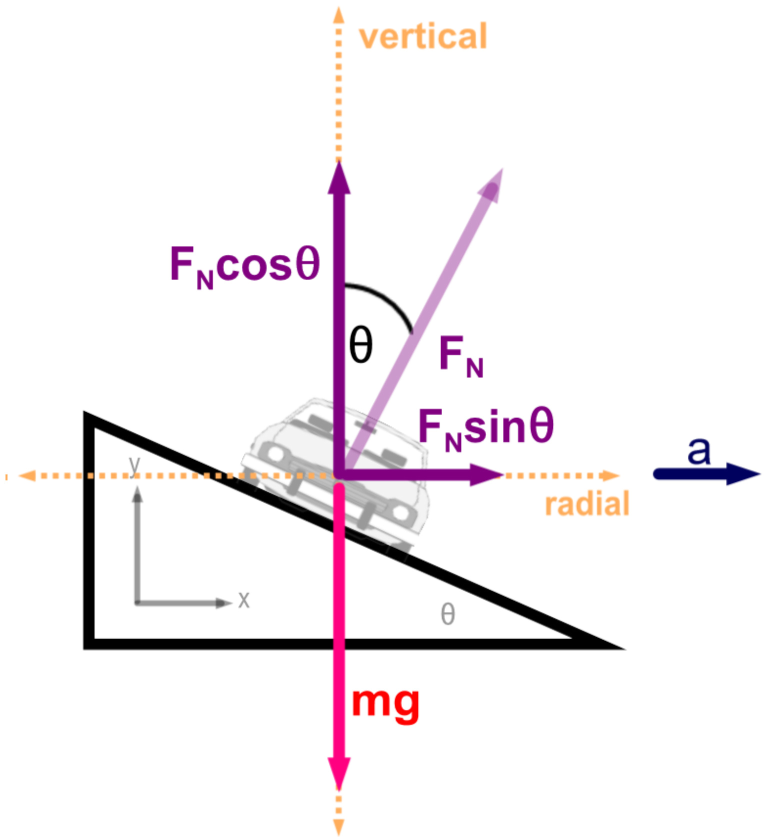
Let's assume that no friction is necessary at the speed we are solving for.

Banked Curves



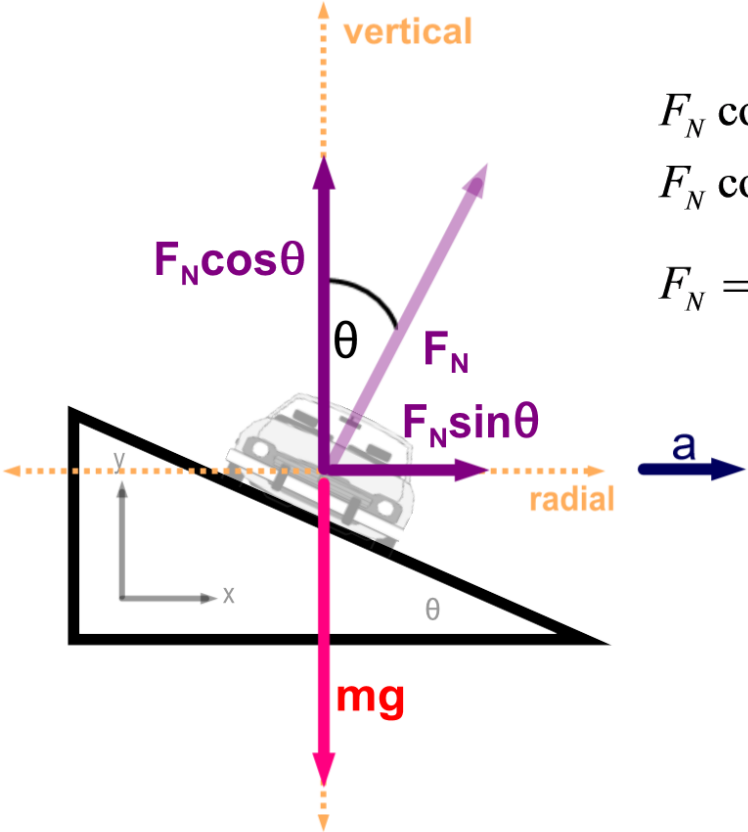
Next, decompose the forces that don't align with an axis, F_N.

Banked Curves



Now let's solve for the velocity such that no friction is necessary to keep a car on the road while going around a curve of radius r and banking angle θ.

Banked Curves



l direction

$$F_N \cos\theta - mg = 0$$

$$F_N \cos\theta = mg$$

$$F_N = \frac{mg}{\cos\theta}$$

radial direction

$$F_N \sin\theta = ma$$

$$F_N \sin\theta = \frac{mv^2}{r}$$

$$\frac{mg}{\cos\theta}\sin\theta = \frac{mv^2}{r}$$

$$g\tan\theta = \frac{v^2}{r}$$

$$v = \sqrt{gr\tan\theta}$$

16 Which of the following is responsible for how a car stays in place on a frictionless banked curve?

- A The vertical component of the car's weight.
- B The horizontal component of the car's weight.
- C The vertical component of the normal force.
- D The horizontal component of the normal force.
- E I need help

Answer

17 Determine the velocity that a car should have while traveling around a frictionless curve with a radius 250 m is banked at an angle of 15 degrees.

- 16.3 m/s
- 19.7 m/s
- 25.6 m/s
- 28.9 m/s
- I need help

Answer

18 Two banked curves have the same radius. Curve A is banked at an angle of 37 degrees, and curve B is banked at an angle of 53 degrees. A car can travel around curve A without relying on friction at a speed of 30 m/s. At what speed can this car travel around curve B without relying on friction?

- A 20 m/s
- B 30 m/s
- C 40 m/s
- D 60 m/s
- E I need help

Answer

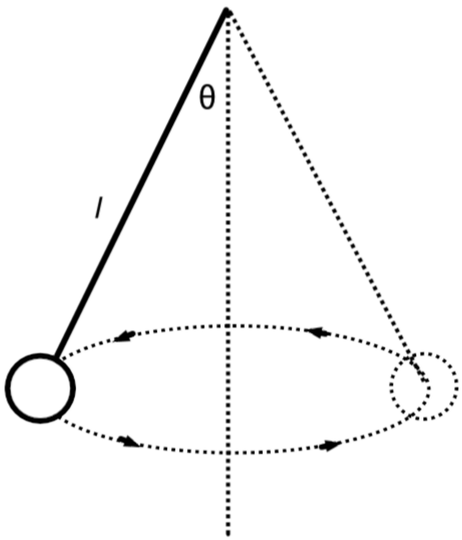
Conical Pendulum

A Conical Pendulum is a pendulum that sweeps out a circle, rather than just a back and forth path.

Since the pendulum bob is moving in a horizontal circle, we can study it as another example of uniform circular motion.

However, it will prove necessary to decompose the forces.

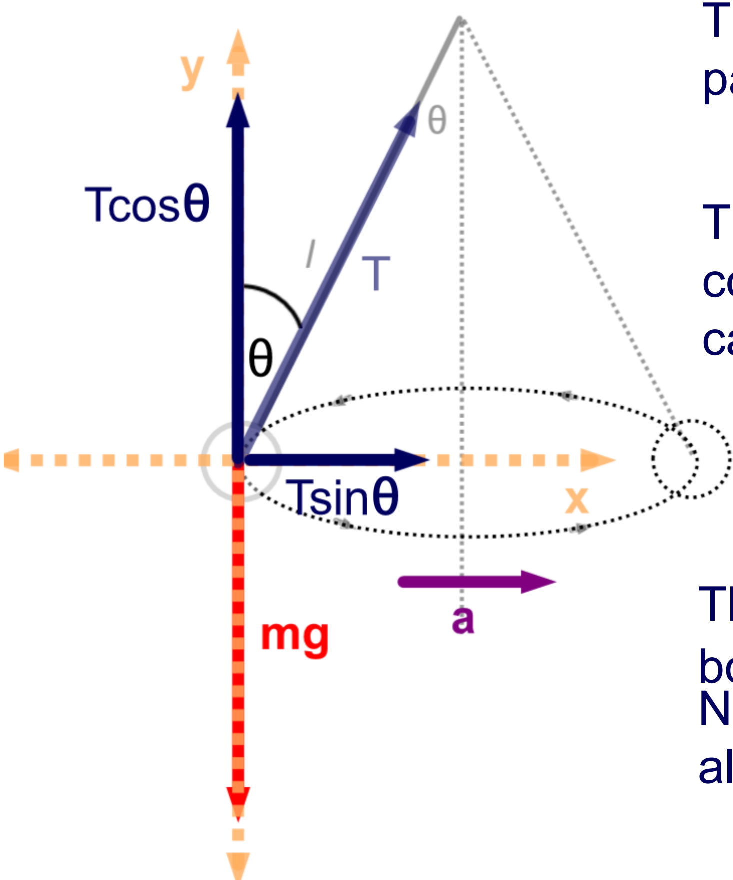
Conical Pendulum



Draw a sketch of the problem, unless one is provided.

Then draw a free body diagram and indicate the direction of the acceleration.

Conical Pendulum



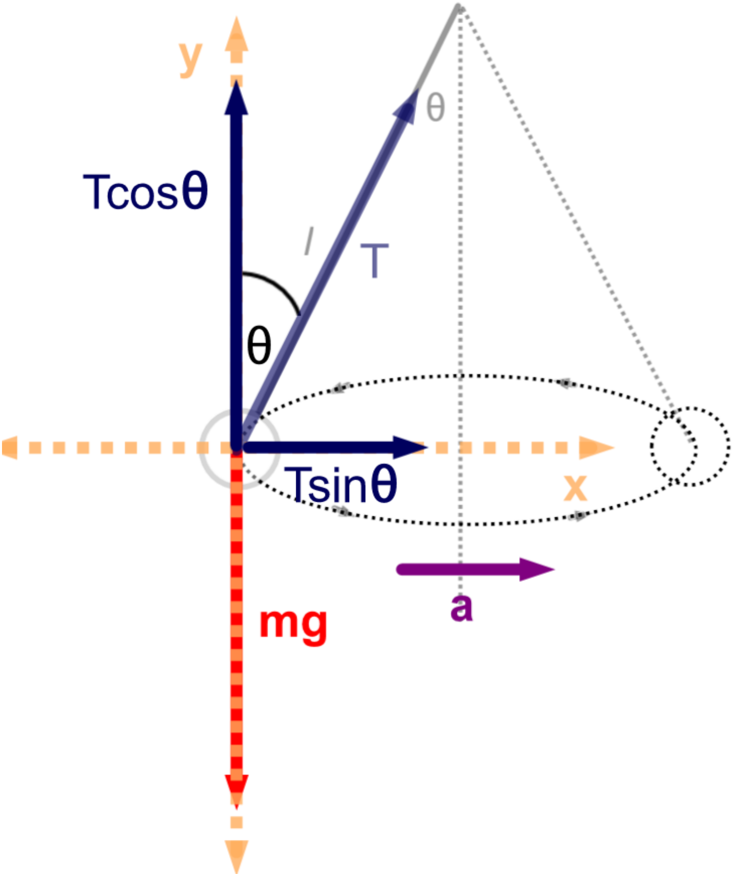
Then draw axes with one axis parallel to acceleration

Then decompose forces so that all components lie on an axis...in this case, decompose T.

Then solve for the acceleration of the bob, based on the angle θ, by applying Newton's Second Law

along each axis.

Conical Pendulum



x - direction

$$T \sin\theta = \frac{mv^2}{r}$$

y - direction

$$T \cos\theta - mg = 0$$

$$T \cos\theta = mg$$

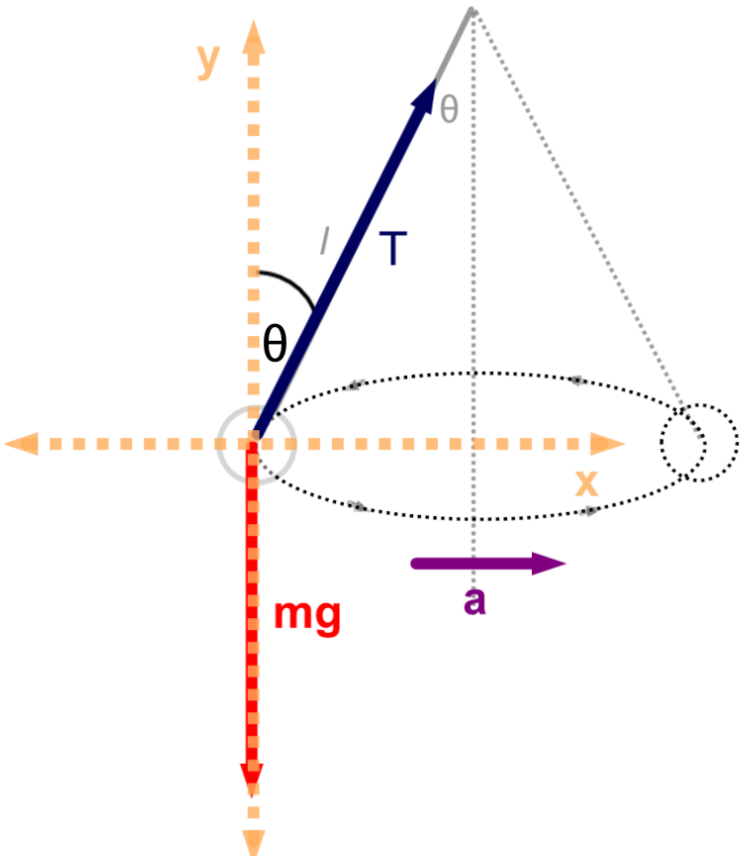
$$T = \frac{mg}{\cos\theta}$$

$$\frac{mg}{\cos\theta} \sin\theta = \frac{mv^2}{r}$$

$$g \tan\theta = \frac{v^2}{r}$$

$$v = \sqrt{gr \tan\theta}$$

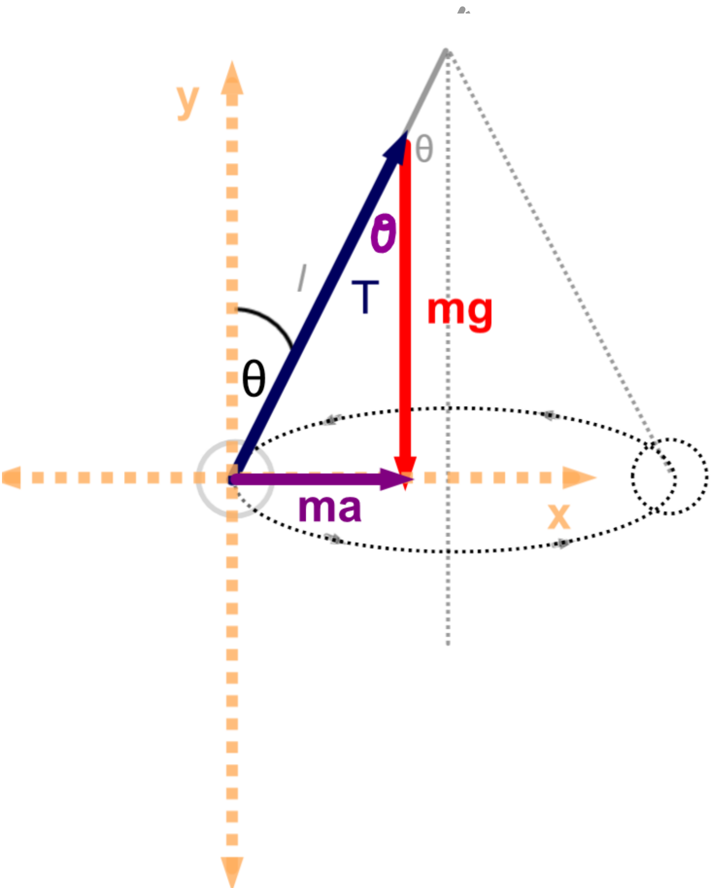
Conical Pendulum



An alternative approach is to solve this as a vector equation using ΣF=ma. (This will work whenever only two forces are present.)

Just translate the original vectors (before decomposing them) to form a right triangle so that the sum of the two forces equals the new vector "ma".

Conical Pendulum



$$\tan\theta = \frac{opp}{adj}$$

$$\tan\theta = \frac{ma}{mg} = \frac{a}{g}$$

$$\tan\theta = \frac{v^2}{gr}$$

$$v = \sqrt{gr\tan\theta}$$

And to find tension....

$$T^2 = (mg)^2 + \left(\frac{mv^2}{r}\right)^2$$

19 A 0.5 kg object is whirled at the end of a rope in a conical pendulum with a radius of 2 m at a speed of 4 m/s. What is the tension in the rope?

- A 2.2 N
- B 6.3 N
- C 7.8 N
- D 9.1 N
- E I need help

Answer

(Short answer) Explain why it is impossible to whirl an object at the end of a string in a horizontal circle.

Answer